AN APPLICATION OF ABSORBING MARKOV CHAINS TO THE ASSESSMENT OF EDUCATION ATTAINMENT RATES WITHIN AIR FORCE MATERIEL COMMAND CIVILIAN PERSONNEL

THESIS

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Abstract

Increasing the education levels of an organization is a common response when attempting to improve organizational performance; however, organizational performance improvements are seldom found when the current and future workforce education levels are unknown. In this research, absorbing Markov chains are used to probabilistically forecast the educational composition of the Air Force Materiel Command civilian workforce to enable organizational performance improvements. Through the purposeful decoupling of effects resulting from recent workforce arrivals and education level progressions, this research attempts to determine the implications that stationarity assumptions have throughout the model development process of an absorbing Markov chain. The results of the analysis indicate that the four combinations of stationarity assumptions perform similarly at representing the historical data and that the forecasted educational attainment rates of the Air Force Materiel Command civilian workforce are expected to increase significantly.

Acknowledgments

I wish to sincerely thank my faculty advisor, Dr. Raymond Hill, for believing in me throughout this process. His expertise, insight, and continual support was instrumental to the success of this effort. I would also like to thank Dr. Ross Jackson and Ms. Amanda Reboulet for their continual support, encouragement, and mentoring. I am indebted to them both for providing me this opportunity. Additionally, I would like to thank Maj Thomas Talafuse for his willingness and ability to discuss modeling difficulties faced. Most importantly, I would like to express my love and admiration for my wife who was patient and understanding throughout my degree program. Her support was, and always will be, an immeasurable force propelling me forward in life.

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AN APPLICATION OF ABSORBING MARKOV CHAINS TO THE ASSESSMENT OF EDUCATION ATTAINMENT RATES WITHIN AIR FORCE MATERIEL COMMAND CIVILIAN PERSONNEL

I. Introduction

1.1 Problem Background

During his first year in office as Air Force Chief of Staff (CSAF), General David Goldfein released a letter to all United States (U.S.) Airmen highlighting adversarial technology investments and their impact in denying the air, space, and cyber superiority that the U.S. has come to rely upon (Goldfein, 2017). A response to this fleeting superiority is that the U.S. must strive to develop a highly skilled and adaptable workforce capable of developing and utilizing technology (Johnson, 1991). However, even if the U.S. can continue developing new technologies, productivity improvements are unlikely unless users also have the educational capability required to operate and handle advanced technologies (Berger, 1987). Further supporting this idea is that the Secretary of the Air Force (SECAF) and Chief of Staff of the Air Force (CSAF), in 2006, stated that advanced education is one of the most effective ways to develop the knowledge and competency required to accomplish the Air Force's mission (Wynne & Moseley, 2006). However, it should be noted that not all organizational tasks within the Air Force require advanced education, the robust use of information technologies, or abstract reasoning according to the Office of Personnel Management (OPM) career classifications ("Classification and Qualification," n.d.). Thus, it is critical to develop an understanding of the current and future educational needs and composition of the Air Force to ensure optimal organizational alignment.

Air Force Materiel Command (AFMC) is one of the ten Major Commands (MAJCOM) within the United States Air Force (USAF). The primary responsibilities of AFMC are conducting research and development, testing and evaluation, and providing acquisition management and logistics support services necessary to keep Air Force weapon systems ready for war ("Air Force Materiel Command," 2018). A simplification of these responsibilities is that AFMC represents the business component of the USAF. AFMC is primarily a civilian command as its workforce composition consists of 80% civil servants and 20% military (Allen, 2017). Therefore, AFMC is business-like in terms of both its responsibilities and workforce composition. As a result, a probabilistic model of the future educational composition of the AFMC workforce could provide relevant comparisons to the educational compositions of corporations.

1.2 Problem Statement

A probabilistic forecast of educational composition depends upon the qualifying assumptions pertaining to model development associated with the forecast. These assumptions are highly influential when attempting to ensure optimal organizational alignment. This thesis develops and compares methods for constructing Markov chains to assess AFMC's educational composition with respect to personnel arrival and transitional proportionment stationarity. This research focuses on creating a tailorable process useful to entities across the USAF, as well as corporations, to attain organizational benefits based on educational initiatives.

1.3 Research Objectives

Decomposing the problem statement yields several key research objectives.

1. How is the educational composition of AFMC civil servants expected to change from the current educational composition?
2. How do assumptions pertaining to personnel arrival and transitional proportionment stationarity effect the forecasted educational composition?
3. How can the analytical results be applied to other USAF entities to provide additional organizational benefit?

1.4 Thesis Overview

The remainder of this thesis is organized into five chapters. Chapter 2 provides a literature review regarding applications of Markov chains within military and educational assessments. Chapter 3 discusses the methodologies for constructing and implementing the Markov chain models. Chapter 4 applies the methodologies to available AFMC civilian employment data. Chapter 5 summarizes the key research findings, addresses research limitations, and identifies additional areas for future work.

II. Literature Review

2.1 Overview

This chapter reviews the relevant literature pertaining to this research. Specifically, the review encompasses educational applications of Markov chains, Markov chain applications within the military, and educational assessments within the military.

A Markov chain is a stochastic process in which movement to the next state depends only on the current state and not the entire past (Tijms, 2003). This “memorylessness” characterization is often referred to as the Markovian property. Movements between system states are called transitions while probabilities associated with those movements are called transition probabilities. In their most basic form, Markov chains are developed and assessed through transition probability matrix development and utilization. These transition probability matrices probabilistically determine how system entities progress between concurrent system states based upon their current distribution among the states.

2.2 Application of Markov Chains to Education

Understanding and assessing the flows of individuals through the educational system is important due to its implications on workforce demands (Mashat, Ragab, & Khedra, 2012). Early literature related to the application of Markov chains to education, both for higher education students and faculty, indicate that student progression and faculty movement can be modeled as a Markov chain because future states are solely dependent on the current state rather than the sequence of preceding states (Bessent & Bessent, 1980; Bleau, 1981; Borden & Dalphin, 1998; Hackett, Magg, & Carrigan, 1999). In recent years, the Markovian approach appears to be one of the most widely used approaches, compared to regression and simulation, in assessing student progression and faculty movement (Symeonaki & Kalamaitanou, 2011). There currently exists three main focus areas of educational applications of Markov chains: modeling student flow through a degree program, modeling student flow through a course of study, and modeling faculty movement through and out of an academic institution.

2.2.1 Modeling Student Flow through a Degree Program

Within the literature of modeling student flow through a degree program using absorbing Markov chains, there exists three sub-focus areas: student enrollment, student progression, and student graduation. Specific to student enrollment, Rahim, Ibrahim, Kasim, and Adnan (2013) developed a Markov chain model to project post-graduate student enrollments to aid in college planning (e.g., faculty requirements). This particular Markov chain model produced results close to the actual data in terms of enrollment counts and detailed information on the students' progress (i.e., average time spent in education system). Additionally, Markov chains have been utilized to assess students' admission and academic performance (Adeleke, Oguntuase, & Ogunsakin, 2014; Brezavšček, Bach, & Baggia, 2017). During model development, Adeleke et al. (2014) used six model states to address four academic levels, withdrawal, and graduation. Similarly, Brezavšček et al. (2017) used seven model states to address three academic levels, withdrawal, graduation, candidacy, and inactive status. In all three of these efforts, fundamental matrices were utilized to calculate the expected time until absorption and the probabilities of absorption (Adeleke et al., 2014; Oguntuase, & Ogunsakin, 2014; Brezavšček et al., 2017; Rahim et al., 2013).

The study of student progression using absorbing Markov chain models has been conducted to determine factors including expected time within various levels of study, duration of candidature, professor workloads, and academic attainment. For example, Al-Awadhi and Konsowa (2010) developed a Markovian model to characterize the students' mean lifetimes in different levels of study in addition to the probability of dropping out of the system. Within the model, graduation, dropping out, and transferring were all classified as absorbing states. Similarly, Mashat et al. (2012) estimated student flow between different levels of study to identify bottlenecks during student transitions from one semester to the next. By using absorbing state classification (i.e., graduation, dropping out, and not-registered), this probabilistic analysis aided in efficiently planning faculty and facility resources. Prior to these two student progression studies, it was shown that Markov chains can also be used to understand the flow of research candidates to help understand the long term underlying probabilities of completion as well as expected durations of candidature (Nicholls, 2007). Nicholls (2007) found that appropriate strategies can subsequently be put in place for the rectification of identified areas of concern. Additionally, Bessent and Bessent (1980) studied the progression of doctoral students to completion to determine if the number of current admissions was creating an undesirable future dissertation workload for supervising professors. More recently, absorbing Markov chains have been applied to student progression while using fuzzy states (Crippa, Mazzoleni, & Zenga, 2016; Symeonaki & Kalamatianou, 2011). The applied fuzzy states, within these probabilistic analyses, were used to express each students' situation as a relational link to present and past academic attainments. This notion of attainment is further developed within the context of this thesis.

Absorbing Markov chains have also been utilized to specifically study student graduation rates and attrition. Borden and Dalphin (1998) studied how student characteristics, such as grade point average and course credit-load, probabilistically effect graduation rates. Their results indicated that there were large, initial differences in graduation rates subject to grade point averages and course credit-loads. Rather than rectification, the purpose of their analysis was issue identification. A similar study was conducted by Al-Awadhi and Ahmed (2002) modeled undergraduate study flow, with emphasis on student attrition, where the model states represented the number of enrolled students and the periods were years of study. This approach contrasts with the various other Markovian approaches up to this point in time where model states were represented by either educational levels or absorptions.

Building off of these previous analyses, Adam (2015) used a Markov chain model to predict the number of graduate students for the coming years. Based on the Mean Absolute Error (MAE) test, the model showed that there was homogeneity and no significant difference when comparing the actual numbers of students with the predicted numbers. This application of prediction and MAE are further developed within the context on this thesis.

The distinguishment between student enrollment, progression and graduation Markovian analyses, as defined above, were based upon the intended purpose of the analysis – not necessarily the entirety of the work performed. With this in mind, additional studies have also been conducted in which the intended purpose was to generally quantify the flow of students through an educational system (Al-Awadhi & Konsowa, 2007; Auwalu, Mohammed, & Saliu, 2013). In summary, these previous analyses, within the context of modeling student flow through a degree program, show significant overlap with one another in terms of both purpose and approach. This overlap suggests a generalizable approach for applying Markov chains to the assessment of predicting educational attainment levels.

2.2.2 Modeling Student Flow through a Course of Study

Markov chains have been used to probabilistically model student flow through a course of study (Hlavatý & Dömeová, 2014; Shah & Burke, 1999). To successfully finish a course, students' progress through various course requirements depends on the completion of previous course requirements. These requirements, and their respective degree of success (i.e., grades), were generally depicted as transient states through the model development processes. Expanding upon this idea, Shah and Burke (1999) assessed the probability of completing an undergraduate course as varying by the age, sex, and field of study of the student. Their model provided estimates for the probability of course completion in addition to the mean time a student takes to complete a course. Similarly, Hlavatý & Dömeová (2014) showed how students' achievements during the previous semester effect their final examination grade through the development of a fundamental matrix. Interestingly, the results suggested that previous semester success likely foreshadows the student's ability of passing or failing their final examination.

2.2.3 Modeling Faculty Movement through and out of an Academic Institution

Markov chains have also been utilized for the assessment of faculty movement through and out of an academic institution (Bleau, 1981; Hackett, Magg, & Carrigan, 1999). Bleau (1981) constructed a Markov chain faculty planning model to describe and better understand the complex phenomena of faculty movement through an institution and its relationship to faculty salary, faculty composition, and faculty turnover rate. The findings of this research suggest that the Markov chain model was a viable means of gaining useful insights and quantitative data on the faculty profile, salary costs, and expected departures. Additionally, the model was found comprehensive and flexible enough to analyze the effects of alternative policies on the faculty composition. While this notion of probable effects of alternative policies are outside the scope of this current thesis, it is recommended for further research pending education attainment results. Hackett et al. (1999) took a contrasting approach intended to quantify the effect of faculty replacement strategies within a college at a research university. Their investigation suggested that a Markovian approach can provide valuable insight when planning for personnel needs in the immediate to ten year future.

2.3 Markov Chain Applications within the Military

Similar to the field of education, Markov chains have been applied to military assessments. There currently exists three main focus areas of Markov chain applications within the military: military manpower planning, offense and defense attrition, and military vehicle availability.

2.3.1 Military Manpower Planning

Military manpower planning aims to minimize the difference between required and available personnel (Abdessameud et al., 2018). Academic journals are replete with Markov chains assessments of civilian manpower planning in terms of both scale and variety (Blakely, 1970; Davies, 1973; Davies, 1981; Nilakantan, 2011; Sales, 1971; Wijngaard, 1983). Similarly, the application of Markov chains for military manpower planning is pervasive and varies greatly across applications and intended purposes.

There have been a good number of Naval – focused studies using Markov chains to include Navy Unrestricted Line Officers (Weber, 1980), Navy Medical Service Corps (Butler, 1990), Indonesian Army officers (Survadi, 1990), Coast Guard Enlisted personnel (Fiebrandt, 1993), Marine Corps first term enlisted personnel (Nguyen, 1997), Navy Nurses (Kinstler, 2005), Army reserve enlisted personnel (Ginther, 2006), Navy Fire Controlmen (McKeon, 2007), Navy Seals (Hooper, 2011), and Marine Corps Acquisition personnel (Nicholson, 2012).

These works, and all subsequent works, were built upon a framework established by Brothers (1974) in which a Markov methodology was first applied to help determine the force structure of the USAF. Brothers' work highlighted the capability of a Markovian model to provide insight for the many tradeoffs and different controls (i.e., recruitment, promotion, and attrition) available to managers of a personnel-based system. Furthermore, he concluded that stability in a personnel-based system and orderly progression could only be accomplished through the establishment of accurate manpower requirements and through proper manpower forecasting. This notion of forecasting, and its implications on educational attainment, is further developed within the context of this thesis. Additionally, specific to the Air Force, in 1984, the RAND Corporation published a report outlining the development and establishment of dynamic Markov chain model used to calculate the probability that an Air Force officer will voluntarily remain in the service based upon various compensation and personnel policies. (Gotz and McCall, 1984). With this in mind, it was found that actual stay/leave decisions of Air Force officers from within the sample period were consistent with the predictions of the developed model.

Within a U.S. Army context, early work by Gass (1991) captured the types of models currently used in production by the military, and the U.S. Army in particular (Gass, 1991). This work also contained a Markov chain model in which the states were representative of various combinations of rank, skill, function, and time-in-service. Each state contained a number of personnel with common attributes at specified points in time.

Zais and Zhang (2015) built a Markov chain model to predict individualized stay/leave decisions within the U.S. Army (Zais & Zhang, 2015). Their work was founded upon the idea that personnel retention is one of the most significant challenges faced by the U.S. Army, and central to this challenge is understanding the incentives of the stay/leave decision for military personnel. Pre-dating this work, and more specific in purpose, Hall (2009) built a Markov chain model to determine the optimal policy regarding when an officer should retire from the U.S. Army (Hall, 2009). Similar to Gass' methodology, Hall's model contained twenty-nine transient states representative of various combinations of grade, years-of-service, and time-in-grade.

In summary, Markov chains lend themselves well to hierarchical manpower systems like those found within the military. Through the defined structure of a

Markovian process, transition and absorption rates can be quantified and predicted between various hierarchical levels. Additional military applications of Markov chains to the assessment of manpower planning include, but are not limited to, questions concerning readiness, capability, and expected times to reach pre-determined benchmarks (Lindquist, 2017; Skulj, Vehovar, & Stamfelj, 2008). In these instances, the structure of the defined transition matrix enabled the prediction of future force structures, given the assumption that future manpower dynamics follow historically observed patterns (Skulj et al.).

2.3.2 Offense and Defense Attrition

In addition to military manpower planning, Markov chains have been utilized to assess attrition occurring during battlefield engagements and attrition of information pertaining to wartime negotiations (Cheng & Moffat, 2012; Nunn et al., 1982; Slantchev, 2003). Nunn et al. (1982) formulated a Markov chain to assess the attrition of a given population of attackers based upon sequential losses. The defenders were modeled with attrition as an independent binomial distribution – where each layer of defense had its own probability. Cheng and Moffat (2012) built upon this Markovian framework by utilizing it in a battle engagement in which both sides produced attrition dependent upon the amount, tactics, and locations of adversarial personnel. This generalizable model was assessed to potentially aid in real-time battlefield conditions for carrying our risk assessments of various proposed plans of action. From an entirely different application, Slantchey (2003) constructed a Markov chain model in which wartime negotiations were comprised of offers and asymmetric information regarding the distribution of power.

Through this model development process, model states use potentially contradictory information from wartime negotiation offers and battlefield intelligence to learn and settle amicably before military victory. In each of these three analyses, Markov chains were used to probabilistically determine the attrition of either personnel or informational value and were assessed to aid in the representation of real-world situations and conflict.

2.3.3 Military Vehicle Availability

Markov chains have also been utilized to assess military vehicle availability (Vasantharaju, Ashok, & Naiju, 2014; Wong, Jefferis, & Montgomery, 2010; Żurek, Borucka, & Ziółkowski, 2016). In the most traditional sense, Żurek et al. modeled military vehicle availability using a Markov chain in which the model states represented usage, standby, maintenance, repair, and standstill in repair. Through this model development process, average durations in each state were calculated as well as the probabilities of remaining in the same state. Aiding in the calculation of military vehicle availability, Wong et al. (2010) studied the probabilistic nature of diesel engine failures. Their Markov chain model was particularly useful in predicting failures, improving maintenance policies, and reducing maintenance costs.

Vasantharaju et al. (2014) used Markov chains to model typical design life cycle processes to explore potential aircraft design time compression. In this analysis, aircraft designs are segmented into sequential states and probabilities are assigned to all foreseeable possibilities. The amalgamation of these probabilities constituted the Markov chains transition probability matrix. The basis of understanding for this analysis was that there exists a clear need to design aircraft in a manner that accommodates futuristic technologies, compresses the design cycle times, and thus increases the number of available aircraft.

2.4 Educational Assessments within the Military

Developing officers with enduring competencies is the key to a strong, responsive, and skilled military force (Staats, Reynolds, & Troxell 2007). A critical component of this development is the formal academic education of officer personnel in appropriate technical disciplines (Etter, 2000). Education is often seen as an integral part of officer development and an indispensable ingredient in initiatives concerning Total Force development (Staats et al., 2007). As a result, numerous educational assessments have been conducted with regard to military personnel.

Military personnel models have a distinct advantage over civilian personnel models because the military is a closed, hierarchical system (Brown, 1999; Merck & Hall, 1971). The military must develop and educate leadership from within the existing personnel pool, which requires strategic foresight and long range planning (Staats et al., 2007). To this effect, a Markovian model with system states characterized by rank was developed by Merck and Hall (1971) to determine the flow of military personnel given grade, years of service, and specialty. The principal attribute of this analysis was its capacity to create future expected values given a starting distribution and a matrix of transitional probabilities (Merck & Hall, 1971). In 1982, a Markov chain model was developed by Rish to specifically analyze annual quotas of advanced, academic degree positions within the civil engineering field. Building upon this narrowly focused study, Deitz (1996) formulated a Markov decision process to determine the minimum number of

Air Force officers that must enter graduate education programs each year in order to satisfy personnel requirements by academic specialty, degree level, and military rank (Dietz, 1996). In addition to describing process behavior, the developed approach also determined the educational policies that would likely satisfy validated personnel requirements at minimum cost.

Markov chains were utilized in these analyses because the dynamic behavior of personnel systems can be effectively described by probabilistic transitions between system states. With similar intent, Brown (1999) refined the Air Force's FORTRAN-based Quota Allocation Model (QuAM) from Deitz (1996) which provided the minimum number of officers, by grade and academic specialty, which must be educated annually to meet the educational needs and requirements of the Air Force. The QuAM's purpose was to satisfy billet requirements, and hence did not consider the overall educational characteristics of the various officer career fields. For example, if the number of billets for a particular career field within the Air Force was reduced, the QuAM would have recommended educating a smaller number of officers. More recently, Jastrzembski (2005) developed the Advanced Academic Degree Inventory Model (AADIM) to employ an inventory management approach to select, educate, and assign officers to duties that require incumbents possessing advanced education in specialized technical disciplines. This AADIM utilized a Markov chain to forecast the educational quotes necessary to achieve the desired advanced education profile within a prescribed period of time.

The RAND Corporation recently published a study in which a probabilistic forecast was developed to more accurately determine the required production level of Air Force officers who earn advanced academic degrees (Terry et al., 2013). This research was motivated by Air Force Education Requirements Board (AFERB) data from FY2000-FY2010 that showed that only 58% of officer assignments to Master's Degree billets and 33% of officer assignments to PhD billets were made such that the officer's degree level and academic specialty matched the billet requirement (Terry et al., 2013). In addition to this quantitative study, the RAND Corporation has also conducted numerous qualitative studies which assess the educational stock of the military as a result of recruiting trends and military education and training (Asch, Kilburn, & Klerman, 1999; Asch & Orvis, 1994; Winkler & Steinberg, 1997).

Department of Defense Instruction (DODI) 1322.10, states that "professional growth opportunities provided via advanced education opportunities are a key incentive for retaining highly qualified officers" (DODI 1322.10, 2008). Kabalar (2003) analyzed the effect of graduate education on promotion to Army Lieutenant Colonel after taking into consideration demographic factors including gender, race, age, marital status, and number of dependents. Similarly, Pearson (2007) used logistic regression to examine the effects of graduate education on the retention of Captains and Majors within the Air Force by considering similar demographic factors and professional characteristics.

2.5 Summary

This chapter discussed relevant literature pertaining to educational applications of Markov chains, Markov chain applications within the military, and educational assessments within the military. The next chapter details the research methodology, which encompasses an overview of the leveraged data structure, absorbing Markov chains, and the development of historical and future composition vectors and transition matrices.

III. Methodology

3.1 Overview

This chapter describes the Markov chain models that were utilized to forecast the educational attainment levels of AFMC civilian employees. The structure of the available data used is discussed along with an overview of absorbing Markov chains. Additionally, this chapter outlines how to develop the composition vector for each year and the transition matrix between each concurrent set of years given the structure of the available data.

3.2 Data Structure

The application of Markov chain estimating techniques require data for states and transitions. Two authoritative sets of data from the OPM were utilized to probabilistically model civilian education attainment rates within AFMC. The two data sets included:

1. AFMC civilian yearly employment counts by highest education attainment and years of service [1998-2017]
2. AFMC civilian yearly accession counts by years or service [2005-2017]

Both of these data sets are housed within data.gov and are open source. The first data set represents yearly snapshots of the current educational composition of the AFMC civilian workforce while the second data set represents the yearly counts of individuals who enter AFMC civil service. Together, these two authoritative sets of historical data provide a means to determine how AFMC civilian employees arrive, transition between education levels, and separate from the system between future concurrent employment compositions.

Several different education level distinctions exist within the data; however, for the purposes of this thesis, they are aggregated into three broad groups: 1) high school degree (comprised of high school degree, below high school degree, and unspecified level of education); 2) bachelor's degree; and 3) advanced degree (comprised of master's degree and doctorate degree). This aggregated data are contained within Appendices A-D.

Aggregated educational labeling for the absorbing Markov chain states represent the highest attained education level of AFMC civilian employees. For example, if an employee has both a bachelor's degree and high school degree, they are represented by the bachelor's degree state. The advanced degree category does not account for the possibility of having multiple advanced degrees (e.g., a person with two master's degrees).

3.3 Absorbing Markov Chains Overview

A Markov chain is a stochastic process in which the next state depends only on the current state and not the entire past (Tijms, 2003). This "memorylessness" characterization is often referred to as the Markovian property. In its most basic form, a Markov chain is developed and assessed through transition matrix development and utilization. This transition matrix captures how entities transition from the current state (i) to the next state (j).

The first step in formulating a discrete-time, absorbing Markov chain model is the identification of a discrete state space (i.e., a set of possible states which characterize an individual entity at a point in time). This discrete state space is divided into two categories: transient states or absorbing states (Tijms, 2003). A transient state refers to a state where an entity moves from state *i* during one period to state *j* in the next period. An absorbing state refers to a state from which an entity does not leave. Five states were employed for the purposes of this thesis: accession, high school degree, bachelor's degree, advanced degree, and separation. Of the five states modeled, only the "separation" state is classified as an absorbing state. This representation, to include the eleven transitional arcs that exist, is shown in Figure 1.

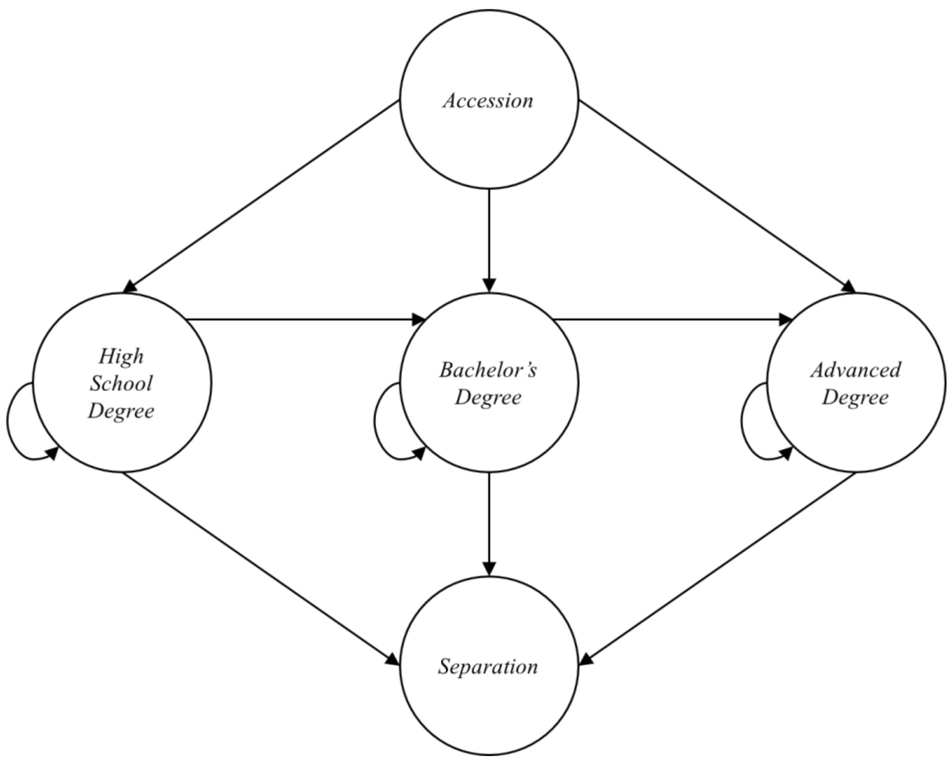

Figure 1. Five-State Absorbing Markov Chain to Assess AFMC Education Attainment

Based upon this formulation, if the AFMC civilian workforce is modeled as a closed-system, all entities would eventually transition into the “separation” state. However, the AFMC civilian workforce is not a closed system (i.e., people can arrive into the system every year). To accurately depict this relationship requires yearly composition vectors, also commonly referred to as alpha vectors, to assess the effect of newly injected personnel into the AFMC civilian workforce. Composition vectors for each year capture the provided AFMC civilian employment counts, by years of service and highest education attainment. Model transition matrices capture AFMC civilian personnel transition among states each year.

3.4 Historical Composition Vector and Transition Matrix Development

Based upon the representation depicted in Figure 1 and the availability of historical data, twelve composition vectors (α_i), and twelve transition matrices (P_i) for the years i = 2005, 2006, …, 2016 were constructed. The composition vectors capture the yearly snapshots of the current education composition of the AFMC civilian workforce while the transition matrices probabilistically model transitions between concurrent employment compositions. As a result, employment information was successively modeled using the following relationship:

$$\alpha_i * P_i = \alpha_{i+1,2-4}. \qquad (1)$$

To model the successive relationship probabilistically using absorbing Markov chains and the provided historical data, the following assumptions were made:

1. The proportion of AFMC civilian employees who separate is distributed evenly across education level; and
2. The proportional breakout of accession counts within each educational level in year i is represented by AFMC civilian employees with less than one year of service in year $i + 1$

The contents for the composition vectors follow (2) as and defined in Table 1:

$$\alpha_i = [\alpha_{i,1} \quad \alpha_{i,2} \quad \alpha_{i,3} \quad \alpha_{i,4} \quad \alpha_{i,5}]. \tag{2}$$

Table 1. Composition Vector Element Definitions and Representations

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i + 1$	Data set 2 for year $i + 1$
$\alpha_{i,2}$	Number of employed AFMC civilians with at maximum a high school degree in year i	Data set 1 for year i
$\alpha_{i,3}$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	Data set 1 for year i
$\alpha_{i,4}$	Number of employed AFMC civilians with at maximum an advanced degree in year i	Data set 1 for year i
$\alpha_{i,5}$	Number of civilians who separated from AFMC between i and $i + 1$	$\alpha_{i,1} + \mathrm{sum}(\alpha_{i,2}, \alpha_{i,3}, \alpha_{i,4}) - \mathrm{sum}(\alpha_{i+1,2}, \alpha_{i+1,3}, \alpha_{i+1,4})$

Table 1 highlights that four of the five elements of the historical compositions vectors are drawn directly from the data. However, the fifth element (i.e., $\alpha_{i,5}$) is calculated as the difference between the current number of civilian employees and the future number of civilian employees after considering the number of people who arrive between the two concurrent years.

The contents of the transition matrices are displayed in (3) and defined in Table 2:

$$P_i = \begin{bmatrix} 0 & P_{i,1,2} & P_{i,1,3} & P_{i,1,4} & 0 \\ 0 & P_{i,2,2} & P_{i,2,3} & 0 & P_{i,2,5} \\ 0 & 0 & P_{i,3,3} & P_{i,3,4} & P_{i,3,5} \\ 0 & 0 & 0 & P_{i,4,4} & P_{i,4,5} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

Table 2. Transition Matrix Element Definitions and Representations

Matrix Element	Definition	Representation/Calculation
$P_{i,1,2}$	Proportion of AFMC civilians with a high school degree and less than 1-year of service in year $i+1$	Data set 1 for year $i+1$
$P_{i,1,3}$	Proportion of AFMC civilians with a bachelor's degree and less than 1-year of service in year $i+1$	Data set 1 for year $i+1$
$P_{i,1,4}$	Proportion of AFMC civilians with an advanced degree and less than 1-year of service in year $i+1$	Data set 1 for year $i+1$
$P_{i,2,5}$	Proportion of AFMC civilians with a high school degree who separated between i and $i+1$	$\alpha_{i,5} \div \mathrm{sum}(\alpha_{i,2}, \alpha_{i,3}, \alpha_{i,4})$
$P_{i,3,5}$	Proportion of AFMC civilians with a bachelor's degree who separated between i and $i+1$	$\alpha_{i,5} \div \mathrm{sum}(\alpha_{i,2}, \alpha_{i,3}, \alpha_{i,4})$
$P_{i,4,5}$	Proportion of AFMC civilians with an advanced degree who separated between i and $i+1$	$\alpha_{i,5} \div \mathrm{sum}(\alpha_{i,2}, \alpha_{i,3}, \alpha_{i,4})$
$P_{i,2,2}$	Proportion of AFMC civilians with a high school degree who self-transition between i and $i+1$	$\left(\alpha_{i+1,2} - \alpha_{i,1}(P_{i,1,2})\right) \div \alpha_{i,2}$
$P_{i,2,3}$	Proportion of AFMC civilians with a high school degree who obtain an bachelor's degree between i and $i+1$	$1 - P_{i,2,2} - P_{i,2,5}$
$P_{i,3,3}$	Proportion of AFMC civilians with a bachelor's degree who self-transition between i and $i+1$	$\left(\alpha_{i+1,3} - \alpha_{i,1}(P_{i,1,3}) - \alpha_{i,2}(P_{i,2,3})\right) \div \alpha_{i,3}$
$P_{i,3,4}$	Proportion of AFMC civilians with a bachelor's degree who obtain an advanced degree between i and $i+1$	$1 - P_{i,3,3} - P_{i,3,5}$
$P_{i,4,4}$	Proportion of AFMC civilians with an advanced degree who self-transition between i and $i+1$	$1 - P_{i,4,5}$

Table 2 highlights that only three non-zero or non-one elements of the historical transition matrices are drawn directly from the data. The other eight non-zero or non-one elements are calculated from those elements drawn directly from the data – either within the historical composition vectors or transition matrices.

3.5 Developing Future Composition Vectors and Transition Matrices

Forecasted educational attainment rates for civilian employees within AFMC are the result of future composition vector and transition matrix development. These future composition vectors and transition matrices are constructed in four different ways based upon the four combinations of arrival and transitional stationarity assumptions:

1. Stationary arrivals and stationary transitions (SA&ST);
2. Stationary arrivals and non-stationary transitions (SA&NT);
3. Non-stationary arrivals and stationary transitions (NA&ST); and
4. Non-stationary arrivals and non-stationary transitions (NA&NT).

AFMC civilian education attainment is forecast out to 2030 based upon the thirteen years of available data for both employment and accession counts. However, a forecast of the total number of AFMC civilian employee arrivals is needed before the stationarity assumptions can be varied for future year educational attainment.

Similarly, forecasts are constructed for the educational arrival proportions and the self-transition proportions depending on the specific combination of stationarity assumptions. For each of these three forecasts, to include future arrivals, the selected forecasting technique is selected from exponential, linear, logarithmic, polynomial or power forecast models.

3.6 Summary

This chapter detailed the research methodology, which encompassed an overview of the leveraged data structure, the absorbing Markov chain model, and the development of the historical and future composition vectors and transition matrices. The next chapter discusses how these methodologies are applied to examine how qualifying assumptions with respect to arrival and transitional proportionment stationarity effect forecasted AFMC education attainment rates.

IV. Analysis and Results

4.1 Overview

This chapter analyzes the Markov chain modeling results associated with the four combinations of stationarity assumptions. This chapter analyzes the work and results from a forecast of AFMC civilian arrivals which was analytically required prior to varying the underlying stationarity assumptions. This chapter compares and contrasts the approaches and findings associated with stochastically modeling AFMC civilian education attainment as an absorbing Markov chain.

4.2 Forecasting AFMC Civilian Employment Arrivals

Each of the following four Markov chain modeling approaches, which are outlined in sections 4.3-4.6, require data associated with the total number of expected arrivals into the AFMC civilian workforce. Appendix Table 12 depicts the historical arrival counts from 2005 to 2017. Using this available data, five different forecasting methods were compared to predict future arrival counts: exponential, linear, logarithmic, polynomial, and power. Table 3 provides the statistical measures of how close the data were to the fitted regression lines (i.e., R^2 or variability explained).

Table 3. Variability Explained by Arrival Forecasting Methods

Forecasting Method	General Form	Variability Explained
Exponential	$y = Ae^{Bx} + C$	0.15562
Linear	$y = Ax + B$	0.10838
Logarithmic	$y = Aln(x) + B$	0.10818
Polynomial	$y = \sum_{i=0}^{n} A_i x^i$	0.24072
Power	$y = Ax^B + C$	0.15541

The variability explained by each of these methods is relatively low since time (i.e., year) is a poor indicator of the number of arrivals into the AFMC civilian workforce. There are numerous other factors that can greatly affect the number of arrivals (e.g., military budget, political will, economic growth, etc.). Table 3 identifies the polynomial forecasting method as the most explanatory for the observed variance. However, the general characteristic and shape of the best-fit polynomial equation is not a good representation of the historical data. Specifically, the best-fit polynomial equation shows a sharp decrease in the accession counts for the years that are forecasted, in addition to a sharp decrease from the years prior to the available data. Neither of these trends are expected. This information is shown in Figure 2.

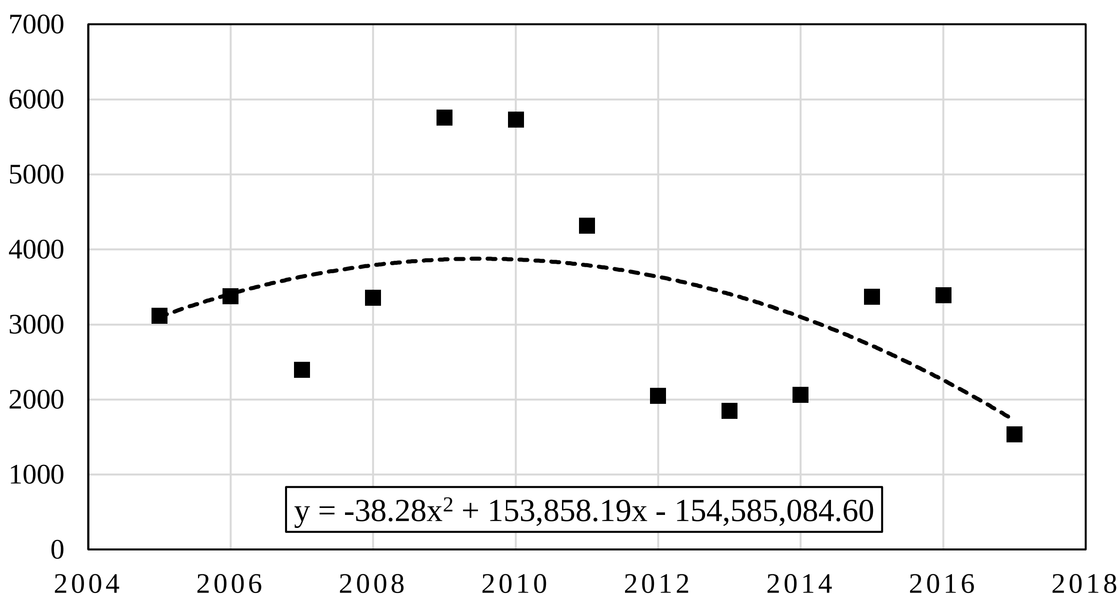

Figure 2. Polynomial Fit of Arrival Counts

Based upon this generalizable lack of fit of the polynomial forecasting method, the exponential forecasting method was next assessed in its ability to forecast the future number of AFMC civilian arrivals. This method explained the second highest amount of variability within the data and was generally well-fit to the historical data. Specifically, the best-fit exponential equation was calculated to have a slightly decreasing slope from the first year of the available data to the last year of the forecast. This decrease was reasonable though given the historically observed trends. This trend is shown pictorially in Figure 3, along with the equation of best fit exponential model for the arrival counts of AFMC civilians.

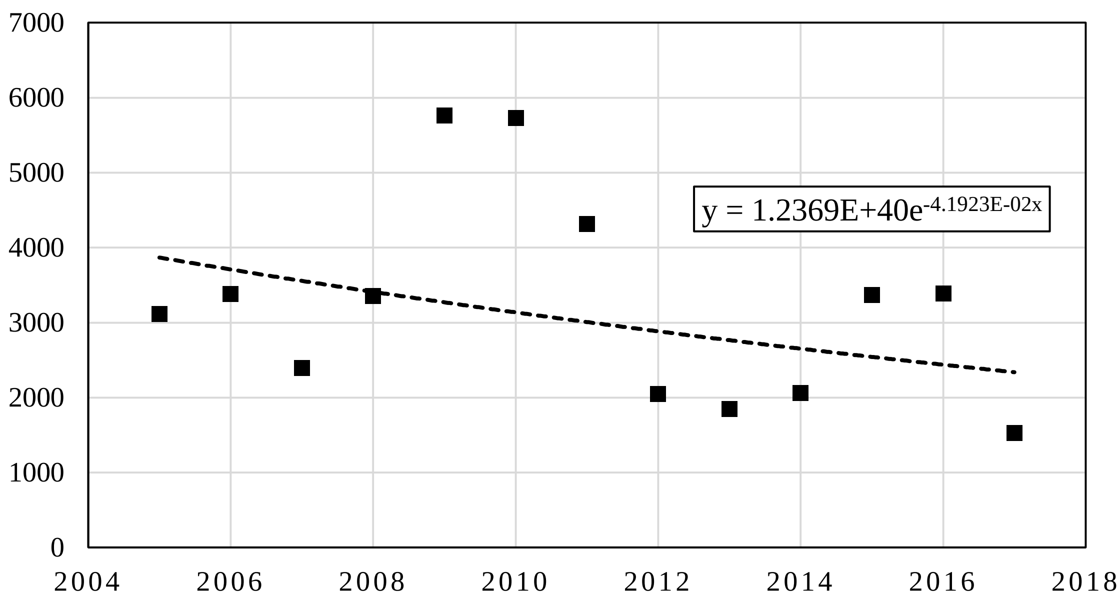

Figure 3. Exponential Fit of Arrival Counts

By utilizing the exponential forecasting method, data was predicted for the years 2018 to 2031 for the number of arrivals in the AFMC civilian workforce. These calculated values are provided in Appendix Table 13. This critical piece of information is used within each of the of the four different Markov chain modeling approaches outlined and discussed below.

4.3 Stationary Arrivals and Stationary Transitions (SA&ST)

This section's results are based on the assumption that both the education arrival proportions (i.e., which educational category a new employee goes into) and the education transition proportions (i.e., the probability of acquiring an additional degree) can be modeled using stationary values. To this effect, two steps were required to determine the six stationary values $(x_1, x_2, \dots, x_6)$. First, the absolute error in the number of people within each education category for the years 2006 to 2017 was minimized by changing the three stationary education arrival proportions (x_1, x_2, x_3). This minimization function is shown in (4) – subject to (5), (6), and (7).

$$Minimize\ Z = \sum_{i=2006}^{2016} \left|\alpha_{i,2} - \alpha_{i,2}^*\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,3} - \alpha_{i,3}^*\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,4} - \alpha_{i,4}^*\right| \quad (4)$$

$$\alpha_i^* * P_i^* = \alpha_{i+1,2-4}^* \quad (5)$$

$$\alpha_i^* = [\alpha_{i,1} \quad \alpha_{i,2}^* \quad \alpha_{i,3}^* \quad \alpha_{i,4}^* \quad \alpha_{i,5}^*] \quad (6)$$

$$P_i^* = \begin{bmatrix} 0 & x_1 & x_2 & x_3 & 0 \\ 0 & P_{i,2,2} & P_{i,2,3} & 0 & P_{i,2,5} \\ 0 & 0 & P_{i,3,3} & P_{i,3,4} & P_{i,3,5} \\ 0 & 0 & 0 & P_{i,4,4} & P_{i,4,5} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Table 4. SA&ST Composition Vector Element Definitions and Representations 1

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i+1$	Data set 2 for year $i+1$
$\alpha_{i,2}^*$	Number of employed AFMC civilians with at maximum a high school degree in year i	$\alpha_{i-1,1}(x_1) + \alpha_{i-1,2}^*(P_{i-1,2,2})$
$\alpha_{i,3}^*$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	$\alpha_{i-1,1}(x_2) + \alpha_{i-1,2}^*(P_{i-1,2,3}) + \alpha_{i-1,3}^*(P_{i-1,3,3})$
$\alpha_{i,4}^*$	Number of employed AFMC civilians with at maximum an advanced degree in year i	$\alpha_{i-1,1}(x_3) + \alpha_{i-1,2}^*(P_{i-1,3,4}) + \alpha_{i-1,3}^*(P_{i-1,4,4})$
$\alpha_{i,5}^*$	Number of civilians who separated from AFMC between i and $i+1$	$\alpha_{i,1} + \text{sum}(\alpha_{i,2}^*, \alpha_{i,3}^*, \alpha_{i,4}^*) - \text{sum}(\alpha_{i+1,2}^*, \alpha_{i+1,3}^*, \alpha_{i+1,4}^*)$

Table 4 further defines the composition vector (6) elements when assuming stationary arrivals and stationary transitions. This table and corresponding transition matrix (7) highlight that the optimization was constructed and executed while the education transition proportions were still indicative of their actual values.

Second, the absolute error in the number of people within each education category for the years 2006 to 2017 was similarly minimized by changing the three stationary transition proportions (x_4, x_5, x_6). This minimization function is shown in (8) – subject to (9), (10), and (11).

$$Minimize\ Z = \sum_{i=2006}^{2016} \left|\alpha_{i,2} - \alpha_{i,2}^{**}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,3} - \alpha_{i,3}^{**}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,4} - \alpha_{i,4}^{**}\right| \quad (8)$$

$$\alpha_i^{**} * P_i^{**} = \alpha_{i+1,2-4}^{**} \quad (9)$$

$$\alpha_i^{**} = [\alpha_{i,1} \quad \alpha_{i,2}^{**} \quad \alpha_{i,3}^{**} \quad \alpha_{i,4}^{**} \quad \alpha_{i,5}^{**}] \quad (10)$$

$$P_i^{**} = \begin{bmatrix} 0 & 0.4776 & 0.3184 & 0.2034 & 0 \\ 0 & x_4 & P_{i,2,3}^{**} & 0 & P_{i,2,5}^{**} \\ 0 & 0 & x_5 & P_{i,3,4}^{**} & P_{i,3,5}^{**} \\ 0 & 0 & 0 & x_6 & P_{i,4,5}^{**} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (11)$$

Table 5. SA&ST Composition Vector Element Definitions and Representations 2

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i+1$	Data set 2 for year $i+1$
$\alpha_{i,2}^{**}$	Number of employed AFMC civilians with at maximum a high school degree in year i	$\alpha_{i-1,1}(0.4776) + \alpha_{i-1,2}^{**}(x_4)$
$\alpha_{i,3}^{**}$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	$\alpha_{i-1,1}(0.3184) + \alpha_{i-1,2}^{**}(P_{i-1,2,3}^{**}) + \alpha_{i-1,3}^{**}(x_5)$
$\alpha_{i,4}^{**}$	Number of employed AFMC civilians with at maximum an advanced degree in year i	$\alpha_{i-1,1}(0.2034) + \alpha_{i-1,2}^{**}(P_{i-1,3,4}^{**}) + \alpha_{i-1,3}^{*}(x_6)$
$\alpha_{i,5}^{**}$	Number of civilians who separated from AFMC between i and $i+1$	$P_{i,4,5}^{**}\left(\mathrm{sum}\left(\alpha_{i,2}^{**}, \alpha_{i,3}^{**}, \alpha_{i,4}^{**}\right)\right)$

Table 6. SA&ST Transition Matrix Element Definitions and Representations

Matrix Element	Definition	Representation/Calculation
$P^{**}_{i,2,5}$	Proportion of AFMC civilians with a high school degree who separated between i and $i+1$	$1 - x_6$
$P^{**}_{i,3,5}$	Proportion of AFMC civilians with a bachelor's degree who separated between i and $i+1$	$1 - x_6$
$P^{**}_{i,4,5}$	Proportion of AFMC civilians with an advanced degree who separated between i and $i+1$	$1 - x_6$
x_4	Proportion of AFMC civilians with a high school degree who self-transition between i and $i+1$	x_4
$P^{**}_{i,2,3}$	Proportion of AFMC civilians with a high school degree who obtain an bachelor's degree between i and $i+1$	$x_6 - x_4$
x_5	Proportion of AFMC civilians with a bachelor's degree who self-transition between i and $i+1$	x_5
$P^{**}_{i,3,4}$	Proportion of AFMC civilians with a bachelor's degree who obtain an advanced degree between i and $i+1$	$x_6 - x_5$
x_6	Proportion of AFMC civilians with an advanced degree who self-transition between i and $i+1$	x_6

Tables 5 and 6 further define the composition vector (10) and transition matrix (11) elements when assuming stationary arrivals and stationary transitions. These tables highlight that the optimization was constructed and executed while the education arrival proportions were set to their calculated stationary values. The results of these two steps are captured within the identified stationary matrix (12).

$$P_i = \begin{bmatrix} 0 & 0.4776 & 0.3184 & 0.2034 & 0 \\ 0 & 0.8835 & 0.0424 & 0 & 0.0741 \\ 0 & 0 & 0.8646 & 0.0612 & 0.0741 \\ 0 & 0 & 0 & 0.9259 & 0.0741 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{12}$$

This stationary transition matrix depicts numerous pieces of representative information regarding the historical AFMC civilian workforce. For example, the proportion of individuals who separate from the workforce can be used to calculate the average time spent as an AFMC civil servant (i.e., 1 / 0.0741 = 13.5 years). The proportions of transitioning between educational categories show that individuals are more likely to obtain an advanced degree while working than they are to obtain a bachelor's degree (i.e., 0.0612 > 0.0424). The arrival proportions show that individuals are more likely to arrive into the workforce with a high school degree (or less) than they are with a bachelor's degree or advanced degree (i.e., 0.4776 > 0.3184, 0.4776 > 0.2034).

The transition matrix must accurately model the historical employment data. If the stationary transition matrix does not accurately model the historical employment data, its application to future years is non-representative. Figure 4 depicts how that total number of AFMC civilian employees within each education category differ when using either the calculated values resulting from the stationary transition matrix or the historical data. The arrival count element ($\alpha_{i,1}$) of the composition vectors for this assessment was based on actual historical values rather than the identified best-fit exponential equation provided in Figure 3.

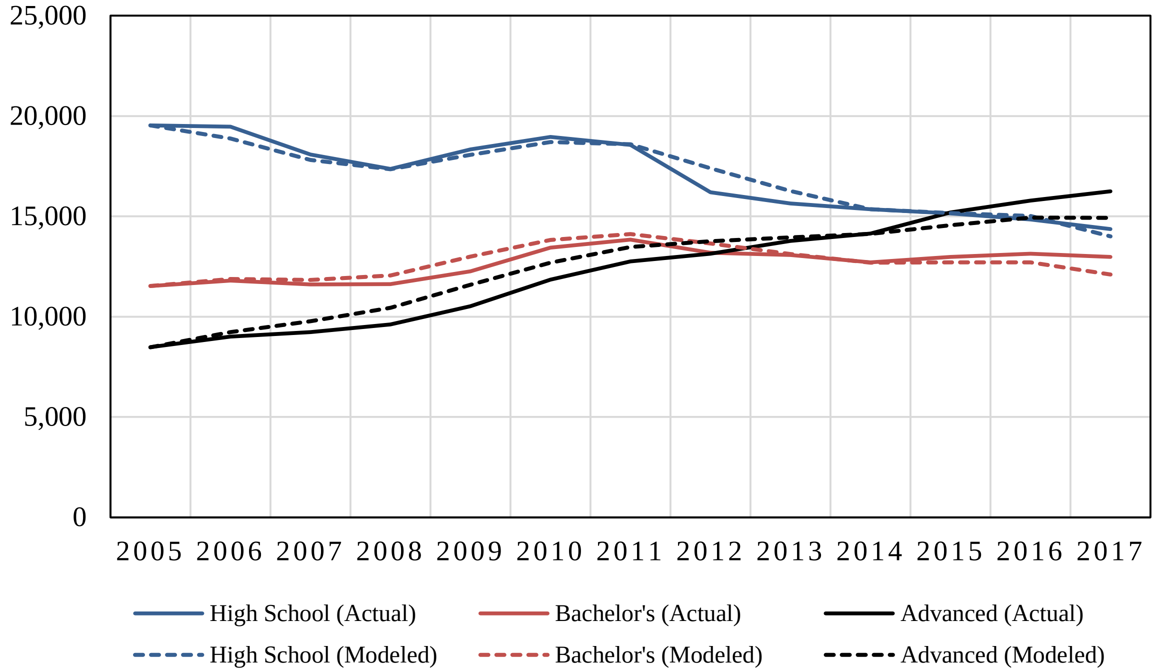

Figure 4. SA&ST Historical Fit

The graphical results in Figure 4 suggest that the stationary transition matrix (12), which resulted from stationary arrivals and stationary transitions assumptions, accurately assesses historical education attainment rates within the AFMC civilian workforce. To objectify this statement, the average relative error of the modeled values was calculated to be 3.31%. Such a relative error is generally considered adequate for estimating purposes (Khair et al., 2017; Lynch & Kim, 2009). Thus, the developed stationary transition matrix was applied, along with the exponentially forecasted count of AFMC civilian arrivals, to assess the future education attainment rates within the AFMC civilian workforce. Figure 5 depicts the result of this application.

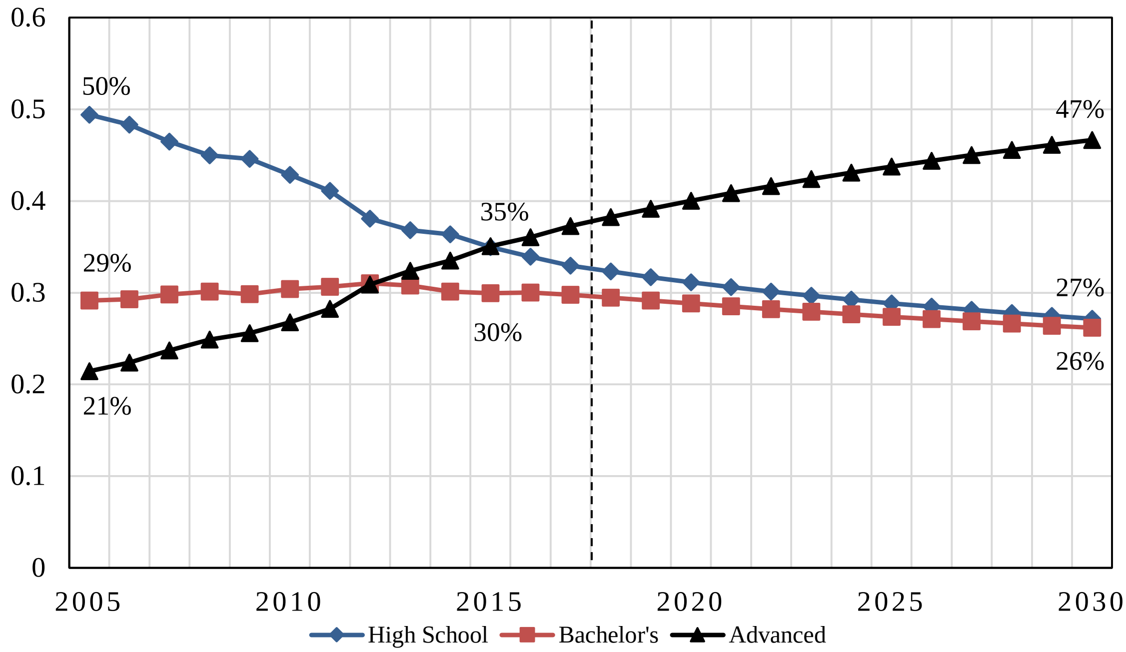

Figure 5. SA&ST Markov Chain Forecast

Combining the composition vectors and the developed stationary transition matrix (12) allows for the calculation of educational attainment proportions. Interestingly, Figure 5 shows that the proportion of AFMC civilians with a bachelor's degree is expected to remain relatively constant at approximately 29%. The proportion of civilians with a high school degree is projected to decline significantly from approximately 50% in 2005, to 27% in 2030. Additionally, advanced degree is expected to undergo a comparably significant increase over the same period of time from 21% to 47%.

4.4 Stationary Arrivals and Non-Stationary Transitions (SA&NT)

This section's results are based on the assumption that the education arrival proportions (i.e., which educational category a new employee goes into) can be modeled using stationary values while the education transition proportions (i.e., the probability of acquiring an additional degree) are modeled using non-stationary values. To this effect, two steps were required to determine the non-stationary transition matrix. First, a forecast was developed to model the self-transition proportions. Five different forecasting methods were compared to predict future self-transition proportions for each education level: exponential, linear, logarithmic, polynomial, and power. The results of this comparison are shown in Table 7.

Table 7. SA&NT Variability Explained by Transition Forecasting Methods

Forecasting Method	High School Degree Variability Explained	Bachelor's Degree Variability Explained	Advanced Degree Variability Explained
Exponential	0.2654	0.1613	0.3648
Linear	0.2771	0.1680	0.3684
Logarithmic	0.2768	0.1678	0.3682
Polynomial	0.3751	0.2815	0.4277
Power	0.2652	0.1611	0.3646

Table 7 identifies the polynomial forecasting method as the most explanatory for the observed variance. However, the general characteristic and shape of the best-fit polynomial equations are not good representations of the historical data. Specifically, the best-fit polynomial equations are heavily influenced by the low self-transition rates in 2011. This information is depicted in Figure 6.

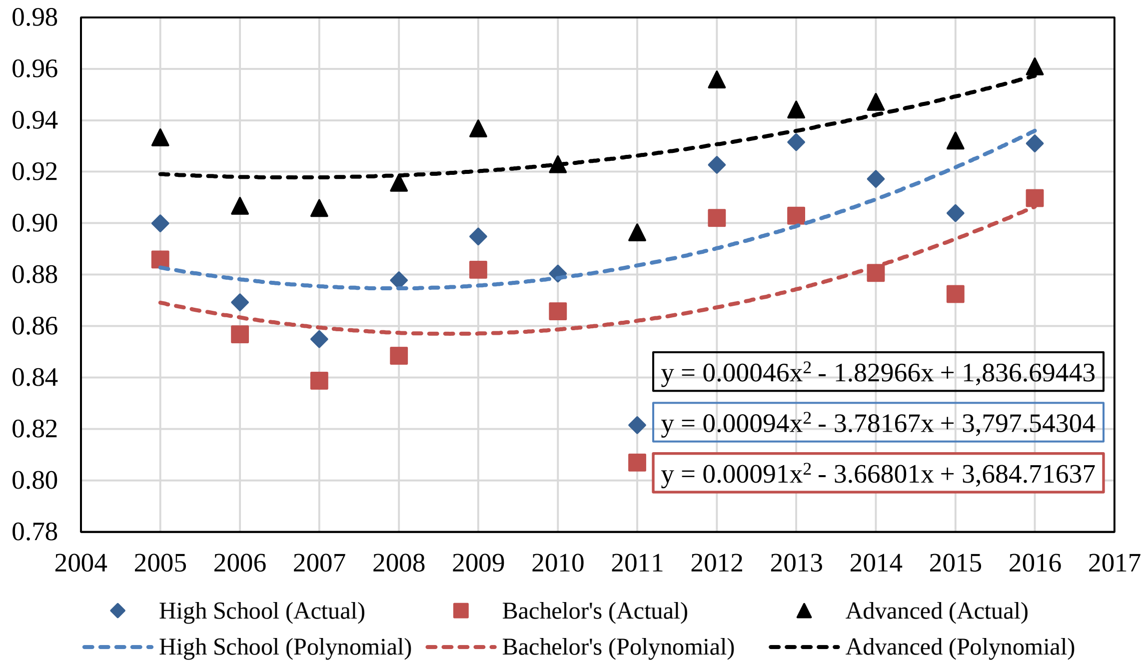

Figure 6. SA&NT Polynomial Fit of Self-Transition Proportions

Based upon this generalizable lack of fit of the polynomial forecasting method, the linear forecasting method was next assessed in its ability to forecast self-transition proportions for each education level. This method explained the second highest amount of variability within the data and was generally well-fit to the historical data. Specifically, the best-fit linear equations were calculated to have a slightly increasing slope from the first year of the available data to the last year of the forecast. This increase was reasonable given the historically observed trends and 2011 data points. These trends are shown pictorially in Figure 7, along with the equations of best fit for the self-transition proportions for each education level.

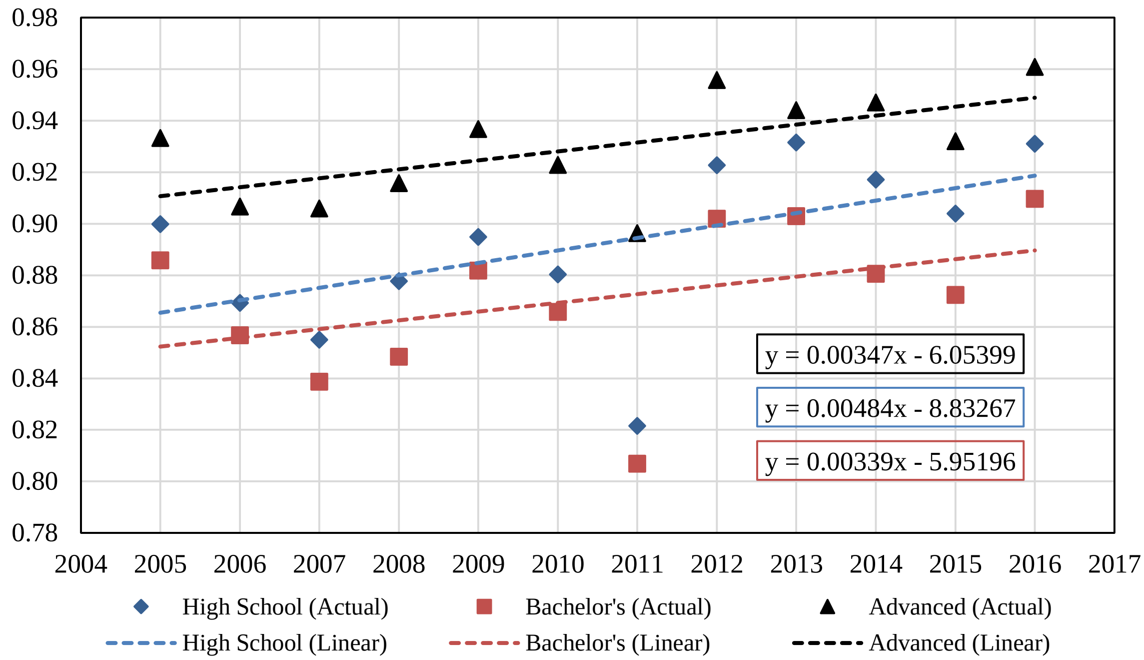

Figure 7. SA&NT Linear Fit of Self-Transition Proportions

Second, the absolute error in the number of people within each education category for the years 2006 to 2017 was minimized by changing the three stationary education arrival proportions (x_7, x_8, x_9). This minimization function is shown in (13) – subject to (14), (15), and (16).

$$Minimize\ Z = \sum_{i=2006}^{2016} \left|\alpha_{i,2} - \alpha_{i,2}^{***}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,3} - \alpha_{i,3}^{***}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,4} - \alpha_{i,4}^{***}\right| \quad (13)$$

$$\alpha_i^{***} * P_i^{***} = \alpha_{i+1,2-4}^{***} \quad (14)$$

$$\alpha_i^{***} = [\alpha_{i,1} \quad \alpha_{i,2}^{***} \quad \alpha_{i,3}^{***} \quad \alpha_{i,4}^{***} \quad \alpha_{i,5}^{***}] \quad (15)$$

$$P_i^{***} = \begin{bmatrix} 0 & x_7 & x_8 & x_9 & 0 \\ 0 & P_{i,2,2}^{***} & P_{i,2,3}^{***} & 0 & P_{i,2,5}^{***} \\ 0 & 0 & P_{i,3,3}^{***} & P_{i,3,4}^{***} & P_{i,3,5}^{***} \\ 0 & 0 & 0 & P_{i,4,4}^{***} & P_{i,4,5}^{***} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

Table 8. SA&NT Composition Vector Element Definitions and Representations

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i+1$	Data set 2 for year $i+1$
$\alpha_{i,2}^{***}$	Number of employed AFMC civilians with at maximum a high school degree in year i	$\alpha_{i-1,1}(x_7) + \alpha_{i-1,2}^{***}(P_{i-1,2,2}^{***})$
$\alpha_{i,3}^{***}$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	$\alpha_{i-1,1}(x_8) + \alpha_{i-1,2}^{***}(P_{i-1,2,3}^{***}) + \alpha_{i-1,3}^{***}(P_{i-1,3,3}^{***})$
$\alpha_{i,4}^{***}$	Number of employed AFMC civilians with at maximum an advanced degree in year i	$\alpha_{i-1,1}(x_9) + \alpha_{i-1,2}^{***}(P_{i-1,3,4}^{***}) + \alpha_{i-1,3}^{***}(P_{i-1,4,4}^{***})$
$\alpha_{i,5}^{***}$	Number of civilians who separated from AFMC between i and $i+1$	$P_{i,4,5}^{***}\left(\text{sum}(\alpha_{i,2}^{***}, \alpha_{i,3}^{***}, \alpha_{i,4}^{***})\right)$

Table 9. SA&NT Transition Matrix Element Definitions and Representations

Matrix Element	Definition	Representation/Calculation
x_7	Proportion of AFMC civilians with a high school degree and less than 1-year of service in year $i+1$	x_7
x_8	Proportion of AFMC civilians with a bachelor's degree and less than 1-year of service in year $i+1$	x_8
x_9	Proportion of AFMC civilians with an advanced degree and less than 1-year of service in year $i+1$	x_9
$P_{i,2,5}^{***}$	Proportion of AFMC civilians with a high school degree who separated between i and $i+1$	$1 - P_{i,4,4}^{***}$
$P_{i,3,5}^{***}$	Proportion of AFMC civilians with a bachelor's degree who separated between i and $i+1$	$1 - P_{i,4,4}^{***}$
$P_{i,4,5}^{***}$	Proportion of AFMC civilians with an advanced degree who separated between i and $i+1$	$1 - P_{i,4,4}^{***}$
$P_{i,2,2}^{***}$	Proportion of AFMC civilians with a high school degree who self-transition between i and $i+1$	Figure 7: High School Linear Equation
$P_{i,2,3}^{***}$	Proportion of AFMC civilians with a high school degree who obtain an bachelor's degree between i and $i+1$	$1 - P_{i,2,2}^{***} - P_{i,2,5}^{***}$
$P_{i,3,3}^{***}$	Proportion of AFMC civilians with a bachelor's degree who self-transition between i and $i+1$	Figure 7: Bachelor's Linear Equation
$P_{i,3,4}^{***}$	Proportion of AFMC civilians with a bachelor's degree who obtain an advanced degree between i and $i+1$	$1 - P_{i,3,3}^{***} - P_{i,3,5}^{***}$
$P_{i,4,4}^{***}$	Proportion of AFMC civilians with an advanced degree who self-transition between i and $i+1$	Figure 7: Advanced Linear Equation

Tables 8 and 9 further define the composition vector (15) and transition matrix (16) elements when assuming stationary arrivals and non-stationary transitions. These tables highlight that the optimization was constructed and executed while the education transition proportions were set to their forecasted non-stationary values. The results of these two steps are captured within the identified non-stationary matrix (17).

$$P_i^{***} = \begin{bmatrix} 0 & 0.3922 & 0.3970 & 0.2109 & 0 \\ 0 & P_{i,2,2}^{***} & P_{i,2,3}^{***} & 0 & P_{i,2,5}^{***} \\ 0 & 0 & P_{i,3,3}^{***} & P_{i,3,4}^{***} & P_{i,3,5}^{***} \\ 0 & 0 & 0 & P_{i,4,4}^{***} & P_{i,4,5}^{***} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

The stationary arrival proportions show that individuals are more likely to arrive into the workforce with a bachelor's degree than they are with a high school degree (or less) or advanced degree (i.e., 0.3970 > 0.3922, 0.3970 > 0.2109). Figure 8 depicts how that total number of AFMC civilian employees within each education category differ when using either the calculated values resulting from this non-stationary transition matrix (17) or the historical data. The arrival count element ($\alpha_{i,1}$) of the composition vectors for this assessment was based on actual historical values rather than the identified best-fit exponential equation provided in Figure 3.

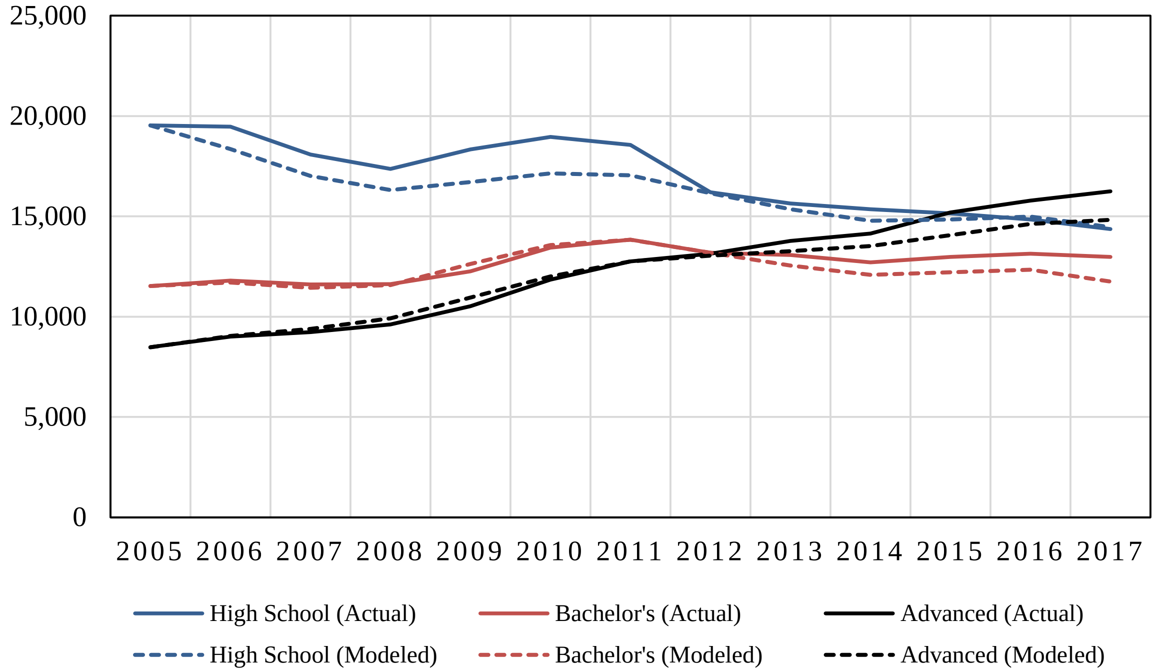

Figure 8. SA&NT Historical Fit

The graphical results in Figure 8 suggest that the non-stationary transition matrix (17), which resulted from stationary arrivals and non-stationary transitions assumptions, accurately assesses historical education attainment rates within the AFMC civilian workforce. To objectify this statement, the average relative error of the modeled values was calculated to be 3.72%. Such a relative error is generally considered adequate for estimating purposes (Khair et al., 2017; Lynch & Kim, 2009). Thus, the developed non-stationary transition matrix was applied, along with the exponentially forecasted count of AFMC civilian arrivals, to assess the future education attainment rates within the AFMC civilian workforce. Figure 9 depicts the result of this application.

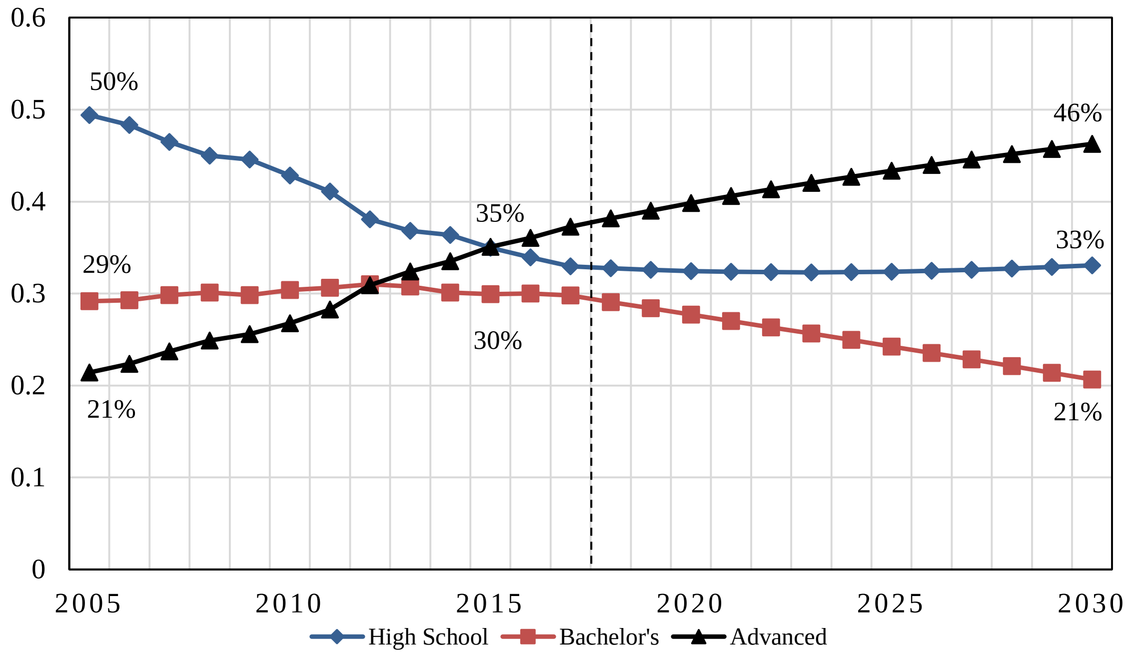

Figure 9. SA&NT Markov Chain Forecast

Combining the composition vectors and the developed non-stationary transition matrix (17) allows for the calculation of educational attainment proportions. Interestingly, Figure 9 shows that the proportion of AFMC civilians with a bachelor's degree is expected to decrease from 29% in 2005, to 21% in 2030. The proportion of civilians with a high school degree is also projected to decrease over the same time period from approximately 50% to 33%. Additionally, advanced degree is expected to undergo a significant increase over the same period of time from 21% to 46%.

4.5 Non-Stationary Arrivals and Stationary Transitions (NA&ST)

This section's results are based on the assumption that the education transition proportions (i.e., the probability of acquiring an additional degree) can be modeled using stationary values while the education arrival proportions (i.e., which educational category a new employee goes into) are modeled using non-stationary values. To this effect, two steps were required to determine the non-stationary transition matrix. First, a forecast was developed to model the education arrival proportions. Five different forecasting methods were compared to predict future arrival proportions for each education level: exponential, linear, logarithmic, polynomial, and power. The results of this comparison are shown in Table 10.

Table 10. NA&ST Variability Explained by Arrival Forecasting Methods

Forecasting Method	High School Degree Variability Explained	Bachelor's Degree Variability Explained	Advanced Degree Variability Explained
Exponential	0.9409	0.6617	0.8877
Linear	0.9275	0.8315	0.9101
Logarithmic	0.9278	0.8320	0.9102
Polynomial	0.9422	0.8656	0.9120
Power	0.9408	0.6624	0.8882

Table 10 results identify the polynomial forecasting method as the most explanatory for the observed variance. This method explained the highest amount of variability within the data and was well-fit to the historical data. These trends are shown pictorially in Figure 10, along with the equations of best fit for the arrival proportions for each education level.

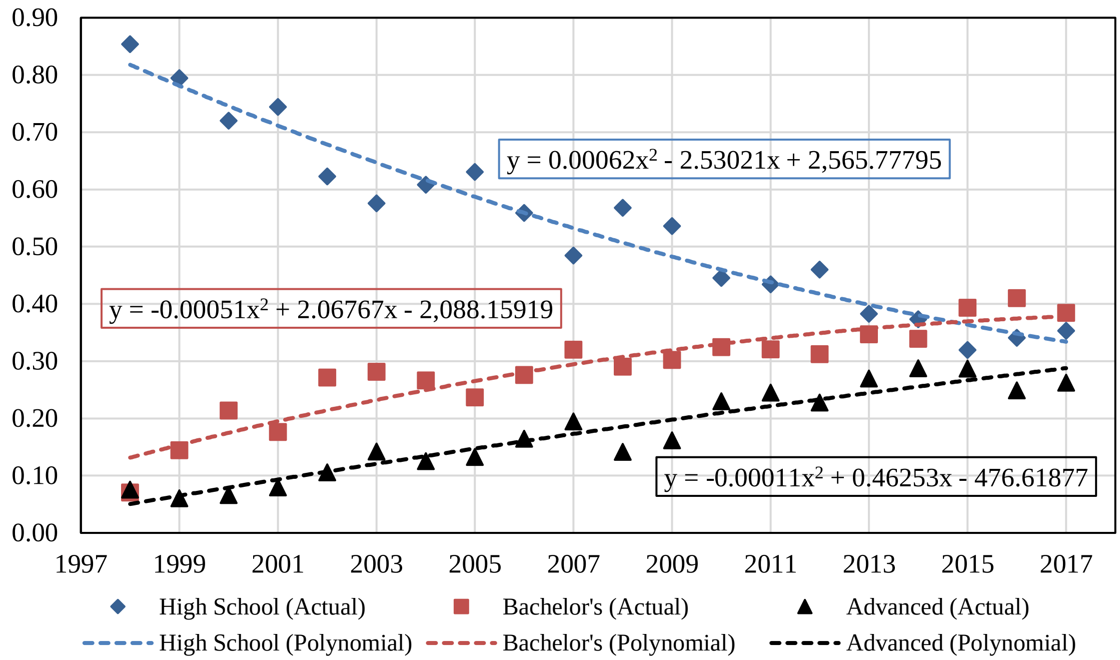

Figure 10. NA&ST Polynomial Fit of Arrival Proportions

Second, the absolute error in the number of people within each education category for the years 2006 to 2017 was minimized by changing the three stationary self-transition proportions (x_{10}, x_{11}, x_{12}). This minimization function is shown in (18) – subject to (19), (20), and (21).

$$Minimize\ Z = \sum_{i=2006}^{2016} \left|\alpha_{i,2} - \alpha_{i,2}^{****}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,3} - \alpha_{i,3}^{****}\right| + \sum_{i=2006}^{2016} \left|\alpha_{i,4} - \alpha_{i,4}^{****}\right| \quad (18)$$

$$\alpha_i^{****} * P_i^{****} = \alpha_{i+1,2-4}^{****} \quad (19)$$

$$\alpha_i^{****} = [\alpha_{i,1} \quad \alpha_{i,2}^{****} \quad \alpha_{i,3}^{****} \quad \alpha_{i,4}^{****} \quad \alpha_{i,5}^{****}] \quad (20)$$

$$P_i^{****} = \begin{bmatrix} 0 & P_{i,1,2}^{****} & P_{i,1,3}^{****} & P_{i,1,4}^{****} & 0 \\ 0 & x_{10} & P_{i,2,3}^{****} & 0 & P_{i,2,5}^{****} \\ 0 & 0 & x_{11} & P_{i,3,4}^{****} & P_{i,3,5}^{****} \\ 0 & 0 & 0 & x_{12} & P_{i,4,5}^{****} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Table 11. NA&ST Composition Vector Element Definitions and Representations

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i+1$	Data set 2 for year $i+1$
$\alpha_{i,2}^{****}$	Number of employed AFMC civilians with at maximum a high school degree in year i	$\alpha_{i-1,1}(P_{i-1,1,2}^{****}) + \alpha_{i-1,2}^{****}(x_{10})$
$\alpha_{i,3}^{****}$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	$\alpha_{i-1,1}(P_{i-1,1,3}^{****})$ $+ \alpha_{i-1,2}^{****}(P_{i-1,2,3}^{****}) + \alpha_{i-1,3}^{****}(x_{11})$
$\alpha_{i,4}^{****}$	Number of employed AFMC civilians with at maximum an advanced degree in year i	$\alpha_{i-1,1}(P_{i-1,1,4}^{****})$ $+ \alpha_{i-1,2}^{****}(P_{i-1,3,4}^{****}) + \alpha_{i-1,3}^{****}(x_{12})$
$\alpha_{i,5}^{****}$	Number of civilians who separated from AFMC between i and $i+1$	$P_{i,4,5}^{****}\left(\mathrm{sum}(\alpha_{i,2}^{****}, \alpha_{i,3}^{****}, \alpha_{i,4}^{****})\right)$

Table 12. NA&ST Transition Matrix Element Definitions and Representations

Matrix Element	Definition	Representation/Calculation
$P_{i,1,2}^{****}$	Proportion of AFMC civilians with a high school degree and less than 1-year of service in year $i+1$	Figure 10: High School Polynomial Equation
$P_{i,1,3}^{****}$	Proportion of AFMC civilians with a bachelor's degree and less than 1-year of service in year $i+1$	Figure 10: Bachelor's Polynomial Equation
$P_{i,1,4}^{****}$	Proportion of AFMC civilians with an advanced degree and less than 1-year of service in year $i+1$	Figure 10: Advanced Polynomial Equation
$P_{i,2,5}^{****}$	Proportion of AFMC civilians with a high school degree who separated between i and $i+1$	$1 - x_{12}$
$P_{i,3,5}^{****}$	Proportion of AFMC civilians with a bachelor's degree who separated between i and $i+1$	$1 - x_{12}$
$P_{i,4,5}^{****}$	Proportion of AFMC civilians with an advanced degree who separated between i and $i+1$	$1 - x_{12}$
x_{10}	Proportion of AFMC civilians with a high school degree who self-transition between i and $i+1$	x_{10}

$P_{i,2,3}^{****}$	Proportion of AFMC civilians with a high school degree who obtain an bachelor's degree between i and $i+1$	$x_{12} - x_{10}$
x_{11}	Proportion of AFMC civilians with a bachelor's degree who self-transition between i and $i+1$	x_{11}
$P_{i,3,4}^{****}$	Proportion of AFMC civilians with a bachelor's degree who obtain an advanced degree between i and $i+1$	$x_{12} - x_{11}$
x_{12}	Proportion of AFMC civilians with an advanced degree who self-transition between i and $i+1$	x_{12}

Tables 11 and 12 further define the composition vector (20) and transition matrix (21) elements when assuming non-stationary arrivals and stationary transitions. These tables highlight that the optimization was constructed and executed while the education arrival proportions were set to their forecasted non-stationary values. The results of these two steps are captured within the identified non-stationary matrix (22).

$$P_i^{****} = \begin{bmatrix} 0 & P_{i,1,2}^{****} & P_{i,1,3}^{****} & P_{i,1,4}^{****} & 0 \\ 0 & 0.8818 & 0.0436 & 0 & 0.0746 \\ 0 & 0 & 0.8613 & 0.0641 & 0.0746 \\ 0 & 0 & 0 & 0.9254 & 0.0746 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (22)$$

This non-stationary transition matrix depicts numerous pieces of representative information regarding the historical AFMC civilian workforce. For example, the proportion of individuals who separate from the workforce can be used to calculate the average time spent as an AFMC civil servant (i.e., 1 / 0.0746 = 13.4 years). The proportions of transitioning between educational categories show that individuals are more likely to obtain an advanced degree while working than they are to obtain a bachelor's degree (i.e., 0.0641 > 0.0436).

Figure 11 depicts how that total number of AFMC civilian employees within each education category differ when using either the calculated values resulting from this non-stationary transition matrix (22) or the historical data. The arrival count element ($\alpha_{i,1}$) of the composition vectors for this assessment was based on actual historical values rather than the identified best-fit exponential equation provided in Figure 3.

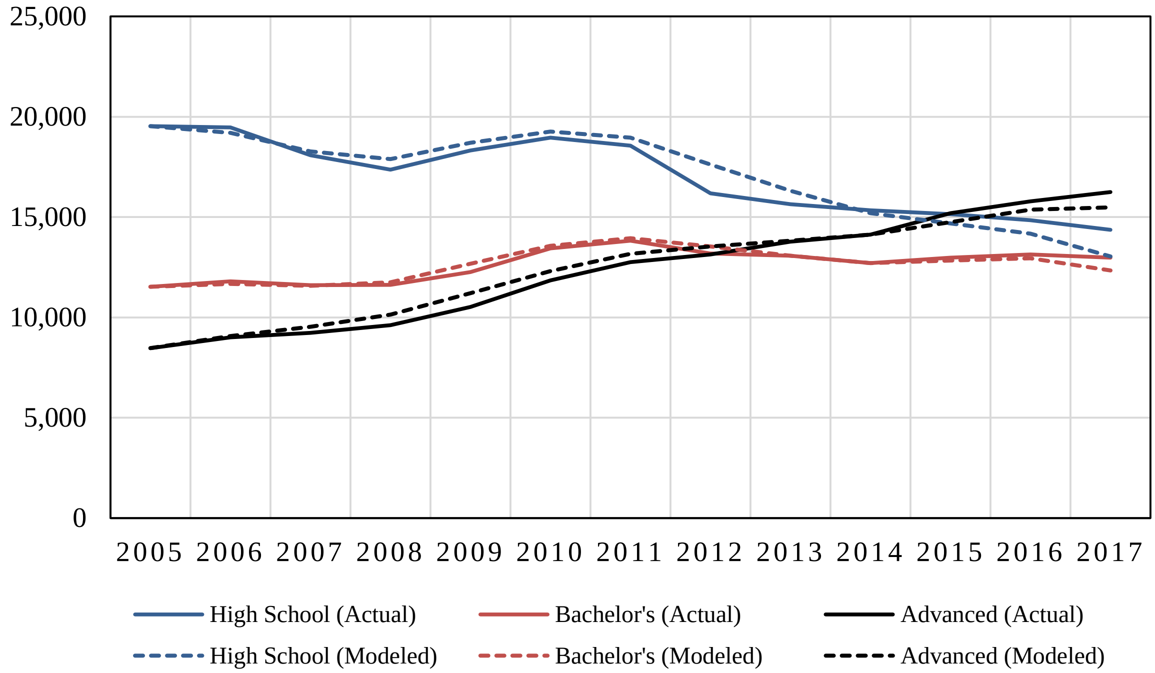

Figure 11. NA&ST Historical Fit

The graphical results in Figure 11 suggest that the non-stationary transition matrix (22), which resulted from non-stationary arrivals and stationary transitions assumptions, accurately assesses historical education attainment rates within the AFMC civilian workforce. To objectify this statement, the average relative error of the modeled values was calculated to be 2.67%. Such a relative error is generally considered adequate for estimating purposes (Khair et al., 2017; Lynch & Kim, 2009). Thus, the developed non-stationary transition matrix was applied, along with the exponentially forecasted count of AFMC civilian arrivals, to assess the future education attainment rates within the AFMC civilian workforce. Figure 12 depicts the result of this application.

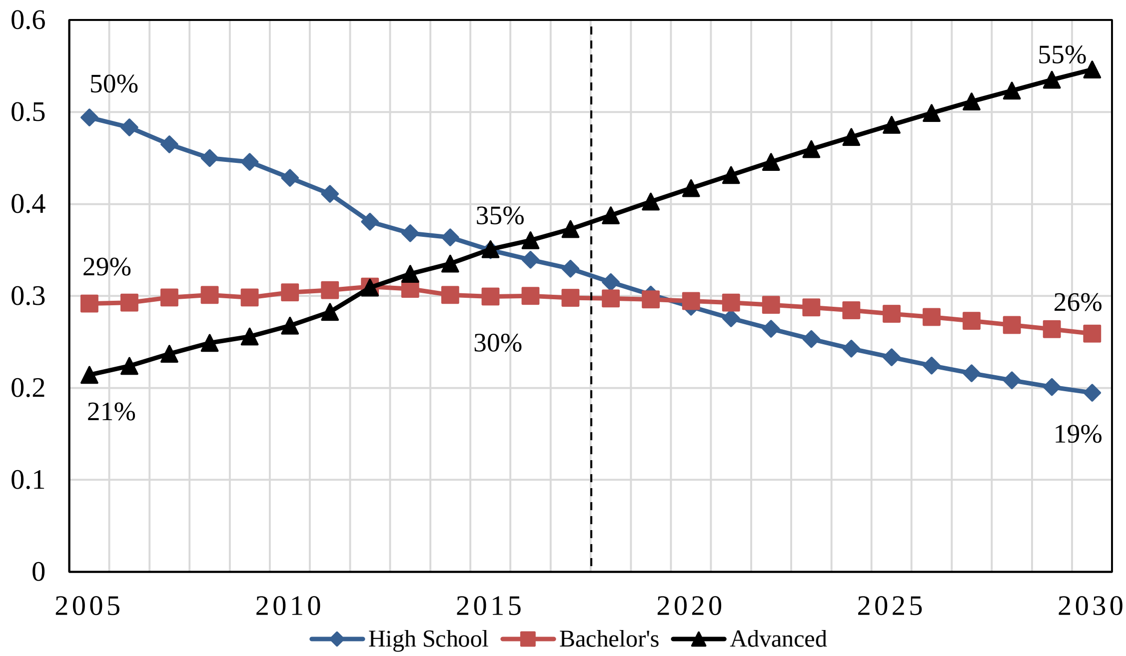

Figure 12. NA&ST Markov Chain Forecast

Combining the composition vectors and the developed non-stationary transition matrix (22) allows for the calculation of educational attainment proportions. Interestingly, Figure 12 shows that the proportion of AFMC civilians with a bachelor's degree is expected to remain relatively constant at approximately 29%. The proportion of civilians with a high school degree is projected to decline significantly from approximately 50% in 2005, to 19% in 2030. Additionally, advanced degree is expected to undergo a comparably significant increase over the same period of time from 21% to 55%.

4.6 Non-Stationary Arrivals and Non-Stationary Transitions (NA&NT)

This section's results are based on the assumption that both the education arrival proportions (i.e., which educational category a new employee goes into) and the education transition proportions (i.e., the probability of acquiring an additional degree) are modeled using non-stationary values. To this effect, two steps were required to determine the non-stationary transition matrix. First, a forecast was developed to model the education arrival proportions. This forecast was previously conducted in section 4.5 and is highlighted in Table 10 and Figure 10. The results showed that the polynomial forecasting method was the most explanatory for the observed variance and was generally well-fit to the historical data.

Second, a forecast was developed to model the education transition proportions. Five different forecasting methods were compared to predict future self-transition proportions for each education level: exponential, linear, logarithmic, polynomial, and power. The results of this comparison are shown in Table 13.

Table 13. NA&NT Variability Explained by Transition Forecasting Methods

Forecasting Method	High School Degree Variability Explained	Bachelor's Degree Variability Explained	Advanced Degree Variability Explained
Exponential	0.17207	0.26834	0.3648
Linear	0.17963	0.2787	0.36846
Logarithmic	0.17935	0.27848	0.36823
Polynomial	0.36992	0.38242	0.42769
Power	0.17179	0.26809	0.36457

Table 13 identifies the polynomial forecasting method as the most explanatory for the observed variance. However, the general characteristic and shape of the best-fit polynomial equations are not good representations of the historical data. Specifically, the best-fit polynomial equations are heavily influenced by the low self-transition rates in 2011. This information is depicted in Figure 13.

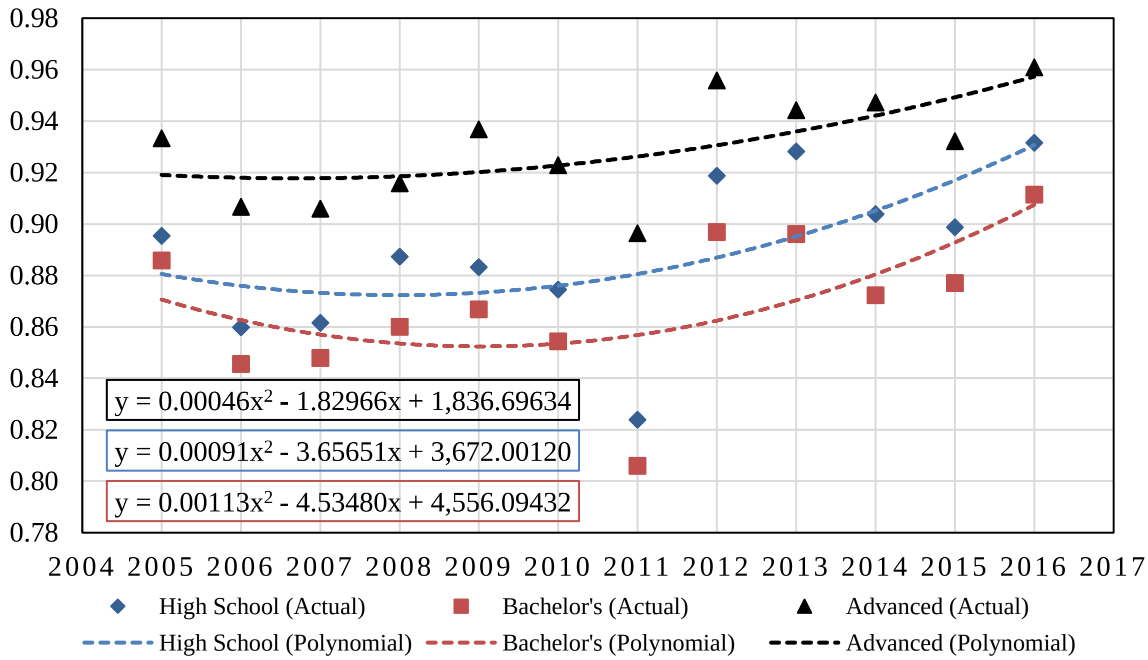

Figure 13. NA&NT Polynomial Fit of Self-Transition Proportions

Based upon this generalizable lack of fit of the polynomial forecasting method, the linear forecasting method was next assessed in its ability to forecast self-transition proportions for each education level. This method explained the second highest amount of variability within the data and was generally well-fit to the historical data. Specifically, the best-fit linear equations were calculated to have a slightly increasing slope from the first year of the available data to the last year of the forecast. This increase was reasonable given the historically observed trends and 2011 data points. These trends are shown pictorially in Figure 14, along with the equations of best fit for the self-transition proportions for each education level.

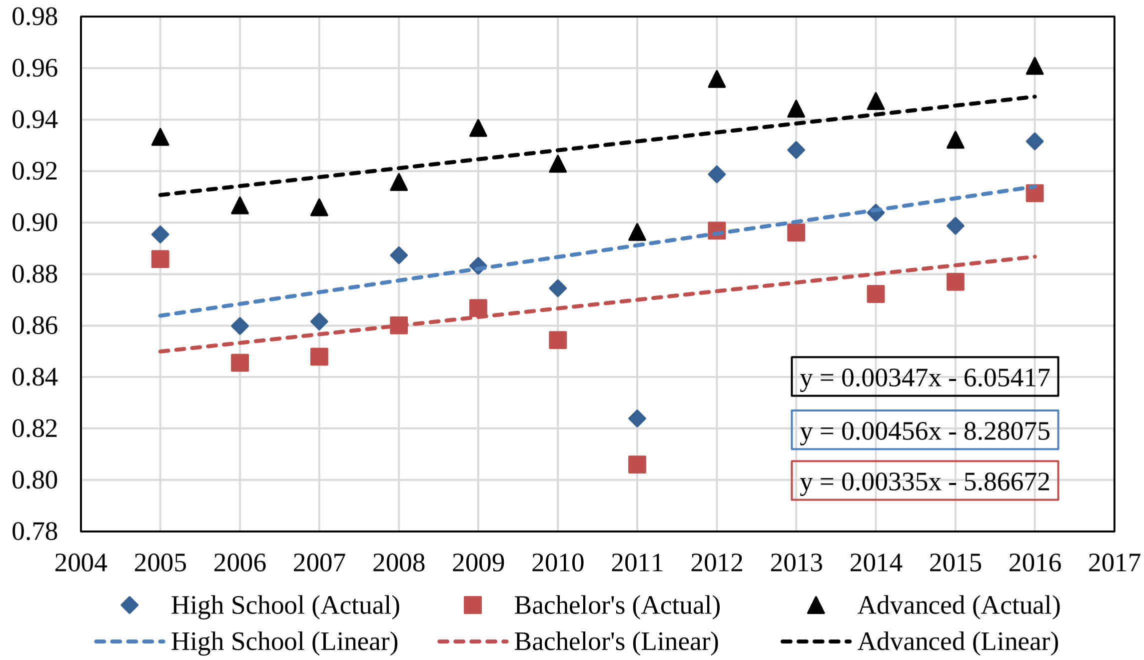

Figure 14. NA&NT Linear Fit of Self-Transition Proportions

Combining the non-stationary forecast from Figure 10 of the arrival proportions and the non-stationary forecast from Figure 14 of the self-transition proportions results in the following composition vector (24) and non-stationary transition matrix (25):

$$\alpha_i^{*****} * P_i^{*****} = \alpha_{i+1,2-4}^{*****} \tag{23}$$

$$\alpha_i^{*****} = [\alpha_{i,1} \quad \alpha_{i,2}^{*****} \quad \alpha_{i,3}^{*****} \quad \alpha_{i,4}^{*****} \quad \alpha_{i,5}^{*****}] \tag{24}$$

$$P_i^{****} = \begin{bmatrix} 0 & P_{i,1,2}^{****} & P_{i,1,3}^{****} & P_{i,1,4}^{****} & 0 \\ 0 & P_{i,2,2}^{*****} & P_{i,2,3}^{*****} & 0 & P_{i,2,5}^{*****} \\ 0 & 0 & P_{i,3,3}^{*****} & P_{i,3,4}^{*****} & P_{i,3,5}^{*****} \\ 0 & 0 & 0 & P_{i,4,4}^{*****} & P_{i,4,5}^{*****} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

Tables 14 and 15 further define the composition vector (24) and transition matrix (25) elements when assuming non-stationary arrivals and stationary transitions. These tables highlight that the subsequent education self-transition forecast was constructed and executed while the education arrival proportions were set to their forecasted non-stationary values.

Table 14. NA&NT Composition Vector Element Definitions and Representations

Vector Element	Definition	Representation/Calculation
$\alpha_{i,1}$	Number of civilians who arrived to AFMC between i and $i+1$	Data set 2 for year $i+1$
$\alpha_{i,2}^{*****}$	Number of employed AFMC civilians with at maximum a high school degree in year i	$\alpha_{i-1,1}(P_{i-1,1,2}^{****}) + \alpha_{i-1,2}^{*****}(P_{i,2,2}^{*****})$
$\alpha_{i,3}^{*****}$	Number of employed AFMC civilians with at maximum a bachelor's degree in year i	$\alpha_{i-1,1}(P_{i-1,1,3}^{****}) + \alpha_{i-1,2}^{*****}(P_{i-1,2,3}^{*****}) + \alpha_{i-1,3}^{*****}(P_{i,3,3}^{*****})$
$\alpha_{i,4}^{*****}$	Number of employed AFMC civilians with at maximum an advanced degree in year i	$\alpha_{i-1,1}(P_{i-1,1,4}^{****}) + \alpha_{i-1,2}^{*****}(P_{i-1,3,4}^{*****}) + \alpha_{i-1,3}^{*****}(P_{i,4,4}^{*****})$
$\alpha_{i,5}^{*****}$	Number of civilians who separated from AFMC between i and $i+1$	$P_{i,4,5}^{*****}\left(\mathrm{sum}\left(\alpha_{i,2}^{*****}, \alpha_{i,3}^{*****}, \alpha_{i,4}^{*****}\right)\right)$

Table 15. NA&NT Transition Matrix Element Definitions and Representations

Matrix Element	Definition	Representation/Calculation
$P_{i,1,2}^{****}$	Proportion of AFMC civilians with a high school degree and less than 1-year of service in year $i+1$	Figure 10: High School Polynomial Equation
$P_{i,1,3}^{****}$	Proportion of AFMC civilians with a bachelor's degree and less than 1-year of service in year $i+1$	Figure 10: Bachelor's Polynomial Equation
$P_{i,1,4}^{****}$	Proportion of AFMC civilians with an advanced degree and less than 1-year of service in year $i+1$	Figure 10: Advanced Polynomial Equation
$P_{i,2,5}^{*****}$	Proportion of AFMC civilians with a high school degree who separated between i and $i+1$	$1-P_{i,4,4}^{*****}$
$P_{i,3,5}^{*****}$	Proportion of AFMC civilians with a bachelor's degree who separated between i and $i+1$	$1-P_{i,4,4}^{*****}$
$P_{i,4,5}^{*****}$	Proportion of AFMC civilians with an advanced degree who separated between i and $i+1$	$1-P_{i,4,4}^{*****}$
$P_{i,2,2}^{*****}$	Proportion of AFMC civilians with a high school degree who self-transition between i and $i+1$	Figure 14: High School Linear Equation
$P_{i,2,3}^{*****}$	Proportion of AFMC civilians with a high school degree who obtain an bachelor's degree between i and $i+1$	$P_{i,4,4}^{*****}-P_{i,2,2}^{*****}$
$P_{i,3,3}^{*****}$	Proportion of AFMC civilians with a bachelor's degree who self-transition between i and $i+1$	Figure 14: Bachelor's Linear Equation
$P_{i,3,4}^{*****}$	Proportion of AFMC civilians with a bachelor's degree who obtain an advanced degree between i and $i+1$	$P_{i,4,4}^{*****}-P_{i,3,3}^{*****}$
$P_{i,4,4}^{*****}$	Proportion of AFMC civilians with an advanced degree who self-transition between i and $i+1$	Figure 14: Advanced Linear Equation

Figure 15 depicts how that total number of AFMC civilian employees within each education category differ when using either the calculated values resulting from this non-stationary transition matrix (25) or the historical data. The arrival count element ($\alpha_{i,1}$) of the composition vectors for this assessment was based on actual historical values rather than the identified best-fit exponential equation provided in Figure 3.

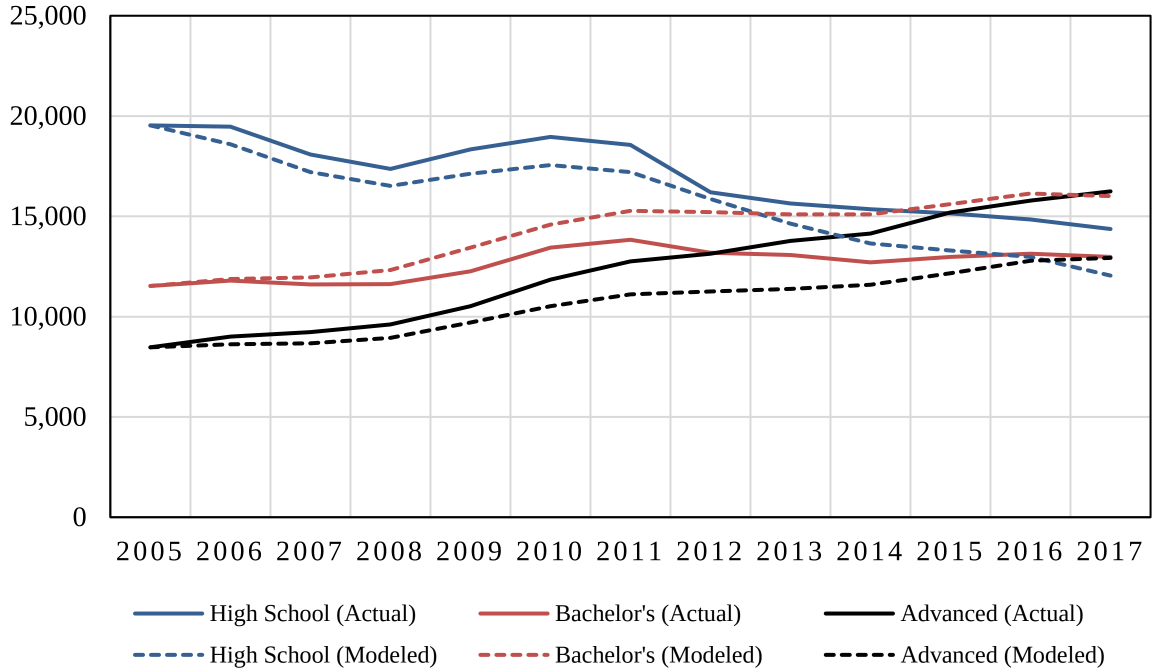

Figure 15. NA&NT Historical Fit

The graphical results in Figure 15 suggest that the non-stationary transition matrix (25), which resulted from non-stationary arrivals and stationary transitions assumptions, does not accurately assesses historical education attainment rates within the AFMC civilian workforce. To objectify this statement, the average relative error of the modeled values was calculated to be 11.33%. Such a relative error is generally not considered adequate for estimating purposes (Khair et al., 2017; Lynch & Kim, 2009). Thus, the developed non-stationary transition matrix was applied, along with the exponentially forecasted count of AFMC civilian arrivals, to assess the future education attainment rates within the AFMC civilian workforce. Figure 16 depicts the result of this application.

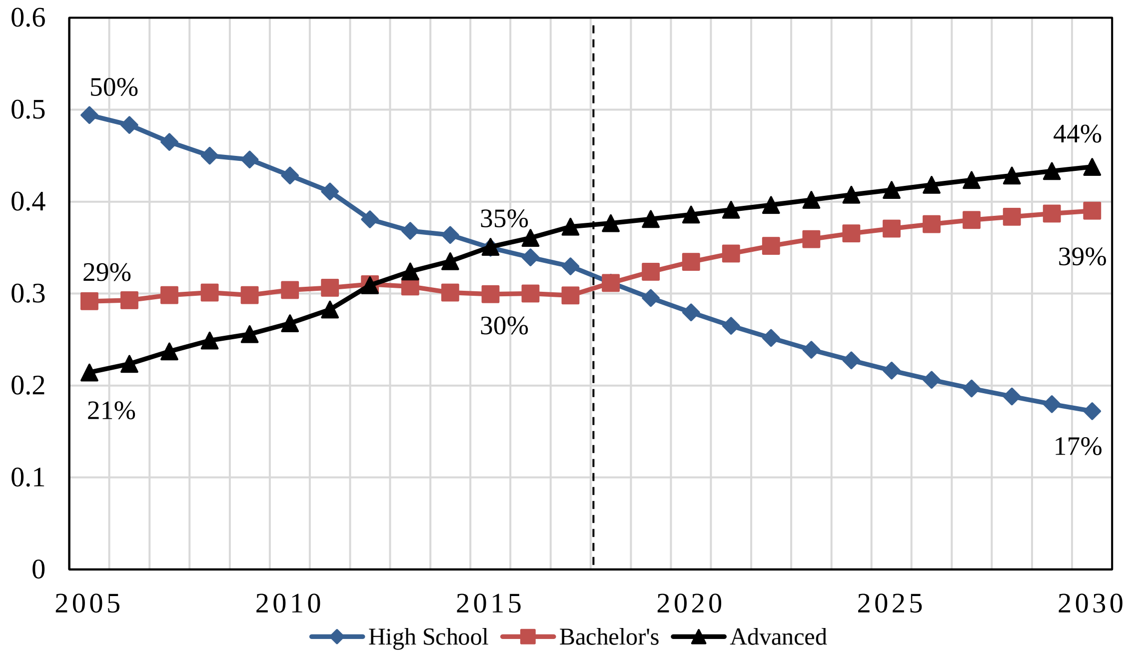

Figure 16. NA&NT Markov Chain Forecast

Combining the composition vectors and the developed non-stationary transition matrix (25) allows for the calculation of educational attainment proportions. Interestingly, Figure 16 shows that the proportion of AFMC civilians with a bachelor’s degree is expected to increase to approximately 39%. The proportion of civilians with a high school degree is projected to decline significantly from approximately 50% in 2005, to 17% in 2030. Additionally, advanced degree is expected to undergo an increase over the same period of time from 21% to 44%.

4.7 Comparison of Methodology Approaches

Each of the four Markov chain modeling approaches, outlined in sections 4.3-4.6, had attributed Mean Average Errors (MAE) associated with interpolating assumed education arrival and education transition functions or values. The functions that were considered for each non-stationary assumption were exponential, linear, polynomial, logarithmic, and power; however, only the polynomial and linear forecasting methods were utilized (across all non-stationarity occurrences). Interestingly, Table 10 depicted that roughly 90% of the variability exhibited in educational arrivals proportions can be accounted for with polynomial interpolations with upward trajectories for both bachelor's and advanced categories and a downward trajectory for the high school category. Sections 4.3-4.6 also identified the forecasted proportions for each educational category for the year 2030. Table 16 summarizes the MAEs and forecasted proportions for each educational category for the year 2030:

Table 16. Summarized AFMC 2030 Forecasted Education Attainment Results

<table>
<tr><th></th><th colspan="3">Education Transition Proportions</th></tr>
<tr><th rowspan="3">Education Arrival Proportions</th><td></td><td>Stationary</td><td>Non-Stationary</td></tr>
<tr><td>Stationary</td><td>SA&ST (4.3), MAE = 3.31%
High School: 27%
Bachelors: 26%
Advanced: 47%</td><td>SA&NT (4.4), MAE = 3.72%
High School: 33%
Bachelors: 21%
Advanced: 46%</td></tr>
<tr><td>Non-Stationary</td><td>NA&ST (4.5), MAE = 2.67%
High School: 19%
Bachelors: 26%
Advanced: 55%</td><td>NA&NT (4.6), MAE = 11.33%
High School: 17%
Bachelors: 39%
Advanced: 44%</td></tr>
</table>

Table 16 can be used to depict numerous key pieces of information regarding the historical AFMC civilian education attainment rates. For example, when assuming stationary education arrivals, the current, relative ordering between high school, bachelors, and advanced degree proportions does not change through the entire forecasted time period from 2018 to 2030. Conversely, when assuming stationary education arrivals, the relative ordering between high school and bachelor's degree proportions swap.

Additionally, Table 16 highlights that assuming non-stationarity, as opposed to stationarity, for the education transition proportions increases the Mean Average Error (MAE). From this information, it can be inferred that the rate in which employees acquire an additional degree has stayed roughly constant since 2005. Additionally, regardless of the stationarity assumptions made during the absorbing Markov chain development process, roughly half of the AFMC civilian workforce is projected to have an advanced degree by 2030.

The MAE for non-stationary arrivals and non-stationary transitions equaled 11.33% which was deemed unfit for forecasting purposes. This result is due largely to the fact that each of the optimization models that were built to identify stationary values had objective functions that aimed to minimize the MAE.

4.8 Summary

This chapter compared and contrasted the findings associated with stochastically modeling AFMC civilian education attainment as an absorbing Markov chain. Specifically, this chapter analyzed how the four combinations of arrival and transition assumptions stationarity assumptions effected forecasted AFMC education attainment rates. The next chapter discusses the key research findings, addresses research limitations, and identifies additional areas for future work.

V. Conclusions and Recommendations

5.1 Research Conclusions

The current education attainment rates of AFMC civilian personnel are approximately evenly proportioned amongst high school degrees, bachelor's degrees, and advanced degrees. This highly education workforce, as compared to other USAF MAJCOMs, Department of Defense organizations, and corporations, represents a potential enabler for workforce capability improvements. Furthermore, recent education trends of the AFMC civilian workforce, from 2005 to 2017, suggest that the number of individuals with advanced degrees will continue to increase while the number of individuals with high school degrees will continue to decrease.

This research used absorbing Markov chains to probabilistically forecast the educational composition of the AFMC civilian workforce. The results indicate that the AFMC civilian workforce is expected to undergo a sizeable increase in education attainment. Specifically by 2030, roughly half of the workforce is expected to have at least one advanced degree. This forecasted value represents the compounded result of two separate model inputs: education arrivals and education transitions. By purposefully decoupling these two model inputs, the four constructed absorbing Markov chain models were afforded the opportunity to calculate various other indicators of workforce performance and health including trends in the average times in service, historical arrival proportions, and historical transition proportions. Regression methods would simply provide point estimates for the overall proportionality of AFMC civilians within each educational level.

The methods and procedures described in this research can similarly be applied to other USAF entities. Additionally, since AFMC is business-like in terms of both its responsibilities and workforce composition, these methods and procedures associated with probabilistically forecasting education attainment can also be applied to corporations. The data required, in either case, includes counts of personnel within each education level for each year, yearly arrivals counts, and education arrival proportions. The results garnered would provide clarity into the current and future organizational composition, which can ultimately influence additionally organizational benefit.

5.2 Limitations

Two key assumptions were made prior to the construction of the absorbing Markov chain models as a result of lacking information in the historical data:

1. The proportion of AFMC civilian employees who separate is distributed evenly across education level; and
2. The proportional breakout of accession counts within each educational level in year i is represented by AFMC civilian employees with less than one year of service in year $i + 1$

The first assumption artificially decreases the number of individuals who separate from higher levels of education. Individuals with advanced degrees, and even those with bachelor's degrees, have greater opportunities in the private sector with sizable increases in salary than those with only a high school degree. Furthermore, one would expect that individuals with more opportunities to separate would do so more often than those individuals with less opportunities. However, the impact that this artificiality has on the results of this research are not as logically defined.

The second assumption does not take into consideration individuals who ‘buy back’ their military time or individuals who enter into civil service but leave within a single year. Arguably, ‘buying back’ military time would increase the arrival proportions for higher levels of education compared to lower levels of education. Although these individuals were correctly accounted for within each of educational categories, they were never labeled as a system arrival. Similarly, the impact that this misappropriation has on the results of this research are not logically defined.

5.3 Future Research

With a newfound understanding of the future educational composition of the AFMC civilian workforce, it is recommended to similarly develop an understanding of the future educational needs to ensure proper organizational alignment. Although advanced education is one of the most effective ways to develop the knowledge and competency required to accomplish the Air Force’s missions, not all organizational tasks within the Air Force require advanced education.

Additionally, this methodology can be applied to subsets of individuals within the AFMC civilian workforce to determine a heightened level of understanding in terms of arrivals, education progression, and separations. For example, Air Force Research Lab civilian personnel, Air Force Life Cycle Management Center civilian personnel, etc. This corresponding analysis would allow decision makers to make more informed decisions regarding hiring best practices, educational incentive programs, and variability in the number of retirements.

Appendix A: Data Set 1 – AFMC Yearly High School Degree Employment Counts

Table 17. 1998-2007 AFMC High School Dimploma Employment Counts

Years of Service	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
< 1 year	982	945	1108	1281	1203	748	774	1056	960	542
1 - 2 years	642	675	690	954	1112	1082	926	928	1130	1113
3 - 4 years	364	433	461	598	683	728	814	949	931	837
5 - 9 years	1694	999	914	986	1114	1140	1285	1598	1899	2107
10 - 14 years	4428	3996	3014	2398	2158	1651	1170	1095	1123	1203
15 - 19 years	5974	5517	5204	4622	4394	3625	3395	2695	2218	1834
20 - 24 years	4629	4529	4620	4825	5060	4436	4311	4399	4043	3352
25 - 29 years	3058	3082	2916	2950	3315	3365	3363	3748	4057	3993
30 - 34 years	2730	2655	2498	2092	1846	1866	1939	2021	2076	2214
> 35 years	692	668	592	871	1118	1049	980	1045	1032	888
TOTAL	25193	23499	22017	21577	22003	19690	18957	19534	19469	18083

Table 18. 2008-2017 AFMC High School Degree Employment Counts

Years of Service	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
< 1 year	972	1555	1930	1226	576	461	543	737	733	537
1 - 2 years	976	1089	2114	2496	1895	1186	769	739	896	1063
3 - 4 years	978	1194	1279	1208	1382	1769	1730	1068	670	632
5 - 9 years	2199	2431	2619	2736	2533	2705	2959	3337	3280	3053
10 - 14 years	1294	1461	1567	1978	2265	2411	2498	2603	2724	2579
15 - 19 years	1557	1175	950	925	918	998	1180	1501	1864	2226
20 - 24 years	2930	2792	2341	1865	1460	1256	855	758	710	740
25 - 29 years	3451	3422	3469	3206	2573	2306	2205	1660	1274	1038
30 - 34 years	2162	2194	1917	2146	1924	1844	1860	1935	1830	1575
> 35 years	848	1017	769	776	666	713	749	815	870	925
TOTAL	17367	18330	18955	18562	16192	15649	15348	15153	14851	14368

Appendix B: Data Set 1 – AFMC Yearly Bachelor’s Degree Employment Counts

Table 19. 1998-2007 AFMC Bachelor's Degree Employment Counts

Years of Service	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
< 1 year	81	172	329	303	524	366	339	397	474	358
1 - 2 years	382	204	280	521	638	864	894	755	801	902
3 - 4 years	250	406	427	289	443	703	798	1025	1046	901
5 - 9 years	1545	864	690	810	875	907	1081	1403	1565	1909
10 - 14 years	3214	3159	2315	1849	1425	1324	834	756	873	925
15 - 19 years	2717	2736	2961	2854	2415	2463	2393	2000	1609	1367
20 - 24 years	1779	1788	1841	1937	1830	2040	2006	2439	2489	2303
25 - 29 years	1405	1465	1362	1303	1248	1337	1260	1432	1618	1699
30 - 34 years	1152	1136	1072	972	807	829	785	844	875	846
> 35 years	403	390	328	359	422	439	425	477	447	393
TOTAL	12928	12320	11605	11197	10627	11272	10815	11528	11797	11603

Table 20. 2008-2017 AFMC Bachelor's Degree Employment Counts

Years of Service	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
< 1 year	497	877	1406	906	391	418	493	908	883	584
1 - 2 years	860	977	1912	2584	2181	1186	797	901	1326	1680
3 - 4 years	961	1074	1235	1300	1915	2544	2022	1121	848	900
5 - 9 years	2124	2259	2315	2401	2302	2482	3070	3725	3701	3491
10 - 14 years	972	1147	1217	1431	1671	1830	1859	1883	1985	1914
15 - 19 years	1210	828	616	688	710	695	810	1019	1193	1409
20 - 24 years	2097	2116	1732	1366	1098	1008	621	489	510	514
25 - 29 years	1692	1701	1936	1980	1775	1657	1702	1345	1057	857
30 - 34 years	820	875	765	882	892	958	988	1200	1233	1173
> 35 years	396	413	312	294	256	299	343	384	401	461
TOTAL	11629	12267	13446	13832	13191	13077	12705	12975	13137	12983

Appendix C: Data Set 1 – AFMC Yearly Advanced Degree Employment Counts

Table 21. 1998-2007 AFMC Advanced Degree Employment Counts

Years of Service	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
< 1 year	87	72	102	137	205	185	160	222	283	218
1 - 2 years	113	145	159	194	218	371	362	394	425	536
3 - 4 years	112	127	169	178	223	315	330	523	676	604
5 - 9 years	901	612	503	495	455	569	624	912	1067	1384
10 - 14 years	1876	1803	1442	1224	871	965	590	668	750	774
15 - 19 years	1644	1733	1884	1891	1601	1792	1561	1576	1414	1236
20 - 24 years	1161	1185	1286	1426	1346	1543	1407	1897	1946	1951
25 - 29 years	1118	1123	1079	941	921	1016	897	1206	1391	1449
30 - 34 years	789	830	833	832	682	714	582	690	652	699
> 35 years	246	262	216	277	291	341	295	383	406	374
TOTAL	8047	7892	7673	7595	6813	7811	6808	8471	9010	9225

Table 22. 2008-2017 AFMC Advanced Degree Employment Counts

Years of Service	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
< 1 year	242	470	997	693	285	325	418	663	536	399
1 - 2 years	519	523	1297	2025	1883	1080	615	798	1153	1269
3 - 4 years	652	784	927	977	1703	2474	2136	1277	856	1012
5 - 9 years	1688	1990	2281	2595	2635	2892	3793	4860	5154	5069
10 - 14 years	854	1060	1156	1337	1762	2127	2297	2581	2918	2983
15 - 19 years	1156	903	700	732	739	779	918	1193	1392	1843
20 - 24 years	1907	1970	1727	1452	1197	1085	733	620	669	709
25 - 29 years	1513	1631	1753	1845	1835	1793	1893	1568	1298	1065
30 - 34 years	718	786	718	844	837	919	1021	1284	1394	1435
> 35 years	363	401	290	260	266	296	310	357	418	461
TOTAL	9612	10518	11846	12760	13142	13770	14134	15201	15788	16245

Appendix D: Data Set 2 – AFMC Yearly Accession Counts

Table 23. 2005-2017 AFMC Accession Counts

Years of Service	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
< 1 year	2343	2413	1654	2499	4252	5053	3146	1670	1341	1657	2588	2480	1119
1 - 2 years	125	159	130	122	195	129	198	130	122	75	158	163	78
3 - 4 years	196	239	185	214	325	108	206	78	111	115	129	133	69
5 - 9 years	215	271	194	272	444	182	334	85	135	122	274	311	141
10 - 14 years	101	122	82	116	220	109	214	24	54	39	101	151	75
15 - 19 years	67	67	61	41	123	51	102	16	32	18	53	53	27
20 - 24 years	32	58	43	39	101	44	65	18	22	14	23	37	8
25 - 29 years	17	20	24	29	56	19	31	12	16	11	18	24	6
30 - 34 years	11	21	11	18	23	22	17	11	11	3	17	25	5
> 35 years	6	10	8	7	21	13	6	6	6	9	10	11	4
TOTAL	3113	3380	2392	3357	5760	5730	4319	2050	1850	2063	3371	3388	1532

Table 24. Forecasted 2018-2031 AFMC Accession Counts

Year	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031
TOTAL	2243	2151	2062	1978	1896	1819	1744	1672	1604	1538	1475	1414	1356	1300

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14. ABSTRACT

Increasing the education levels of an organization is a common response when attempting to improve organizational performance; however, organizational performance improvements are seldom found when the current and future workforce education levels are unknown. In this research, absorbing Markov chains are used to probabilistically forecast the educational composition of the Air Force Materiel Command civilian workforce to enable organizational performance improvements. Through the purposeful decoupling of effects resulting from recent workforce arrivals and education level progressions, this research attempts to determine the implications that stationarity assumptions have throughout the model development process of an absorbing Markov chain. The results of the analysis indicate that the four combinations of stationarity assumptions perform similarly at representing the historical data and that the forecasted educational attainment rates of the Air Force Materiel Command civilian workforce are expected to increase significantly.

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